ION INDUCED DEFORMATION OF SOFT TISSUE

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In this paper the effects of changing the ion concentration in and around a sample of soft tissue are investigated. The triphasic theory developed by Lai et al. (1990, Biomechanics of Diarthrodial Joints, Vol. 1, Berlin, Springer-Verlag) is reduced to two coupled partial differential equations involving fluid ion concentration and tissue solid deformation. These equations are given in general form for Cartesian, cylindrical and spherical geometries. After solving the two equations quantities such as fluid velocity, fluid pressure, chemical potentials and chemical expansion stress may be easily calculated. In the Cartesian geometry comparison is made with the experimental and theoretical work of Myers et al. (1984, ASME J. biomech. Engng, 106, 151–158). This dealt with changing the ion concentration of a salt shower on a strip of bovine articular cartilage. Results were obtained in both free swelling and isometric tension states, using an empirical formula to account for ion induced deformation. The present theory predicts lower ion concentrations inside the tissue than this earlier work. A spherical sample of tissue subjected to a change in salt bath ion concentration is also considered. Numerical results are obtained for both hypertonic and hypotonic bathing solutions. Of particular interest is the finding that tissue may contract internally before reaching a final swollen equilibrium state or swell internally before finally contracting. By considering the relative magnitude, and also variation throughout the time course of terms in the governing equations, an even simpler system is deduced. As well as being linear the concentration equation in the new system is uncoupled. Results obtained from the linear system compare well with those from the spherical section. Thus, biological swelling situations may be modelled by a simple system of equations with the possibility of approximate analytic solutions in certain cases.
1. **Introduction.** When a sample of tissue is placed in a salt bath its dimensions will change according to the concentration of the solution. This is the result of several effects. For example, the ion imbalance between the tissue and the surrounding fluid will cause fluid to be drawn in or expelled. Another reason is that tissue, such as cartilage, contains large molecular chains with a high density of fixed negative charges, referred to here as the fixed charge density (FCD). These repel each other and tend to expand the tissue. An increase in the fluid ion concentration within the medium will shield the charge repulsion and cause the matrix to contract. Conversely, a decrease in ion concentration will allow the tissue to expand. This paper attempts to improve on previous work to describe the time course of deformation, fluid and ion flow, fluid pressure and stress when a soft tissue expands or contracts due to changes in ion concentration.

Previous attempts to model the behaviour of soft tissue, such as cartilage, intervertebral disc and corneal stroma, have stemmed from two directions. The first adopts a physico-chemical viewpoint (Maroudas, 1979; Parsons and Black, 1979) and uses Donnan theory for aqueous polyelectrolyte solutions with no field equations for stress in the solid and ions. The second assumes a biphasic structure composed of a solid, with elastic properties determined by the underlying physico-chemical properties, and a fluid phase (Mow et al., 1980). Later theories, such as those of Lanir (1987), Myers et al. (1984) and Eisenberg and Grodzinsky (1985, 1987) have coupled these ideas to various degrees. For example, Myers et al. (1984) extend the biphasic theory to include a correction term in the solid stress equation to explain swelling due to ion concentration. Eisenberg and Grodzinsky (1987) incorporate the effect of ion concentration on material properties and a chemical stress term. However, the most complete theory to date is the triphasic theory of Lai et al. (1990, 1991) which deals more comprehensively with three phases in the material; an elastic solid which may contain fixed charges and have non-constant material parameters, the water phase and an ionic phase in the form of a salt dissolved in the water. Recently this theory has been applied to the problem of one-dimensional flow through soft tissue (Gu et al., 1993). The complexity of the triphasic theory makes it difficult to implement. The following work is an attempt to reduce it to a tractable form under certain simplifying assumptions, whilst still describing the predominant physical phenomena associated with tissue deformation.

Soft tissue may also be considered as a fibre reinforced gel, hence work on gels and gel diffusion may be relevant, particularly within individual cells. A gel is a cross-linked polymer network immersed in a fluid. The work of Tanaka and Fillmore (1979), and others (e.g. Peters and Candau, 1988; Li and Tanaka, 1989), describe the swelling of gels in water. They obtain a single deformation equation since this problem is independent of charge. The theory presented in this paper reduces to the same equation when the fixed charge density vanishes.

At the beginning of section 2 the reader is referred to appendix A for a