AXISYMMETRIC BENDING FOR THICK LAMINATED CIRCULAR PLATE UNDER A CONCENTRATED LOAD *

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Abstract: Based upon the fundamental equations of three dimensional elasticity, the state equation for axisymmetric bending of laminated transversely isotropic circular plate is established and the concentrated force on plate surface is expanded into Fourier-Bessel’s series, therefore, an analytical solution for the problem is presented. Every fundamental equation of three dimensional elasticity can be exactly satisfied by the solution and all the independent elastic constants can be taken into account fully, furthermore, the continuity conditions between plies can also be satisfied.

Key words: laminated circular plate; state equation; concentrated load; axisymmetric bending

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Nowadays, the current theories of plates and shells, such as Kirchhoff’s thin plate theory and Reissner’s moderately thick plate theory etc., are established on some hypotheses, for example, assuming that the mechanical quantities are the polynomials of a certain coordinate variable \[1,2\]. Ref. [3] has been shown that the true solution for each mechanical quantity can not be a polynomial of any coordinate variable. If the form of a polynomial is adopted, the incompatibility among fundamental equations of elasticity must be brought about in the deductive process and only some of the elastic constants can be taken into account. The errors caused by polynomial hypotheses raise rapidly as the thickness of the plate increases.

Without any assumptions about displacement models and stress distributions, Refs. [3] and [4] take the Fourier functions as basic solution and introduce the theory of state space, the state equations for laminated plates and cylindrical shells with arbitrary thickness under general boundary conditions are established. The solution given satisfies all the fundamental equations of elasticity and the continuity conditions between plies of the laminated plates and shells, but the static solution for circular plate has not been involved. Based upon fundamental equations of three dimensional elasticity, the state equation for laminated transversely isotropic circular plate under cylindrical coordinate system is established by the present paper. Taking Bessel functions as a basic solving function and expanding the concentrated load on plate surface into Fourier-Bessel’s series, the

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solution for the state equation is obtained. By comparing with approximate theory, the results of the present numerical examples are satisfactory and some meaningful conclusions are obtained.

1 Establishment and Solution for the State Equation

Examining a circular plate with transversely isotropic material, the principal elastic directions of the plate coincide with the coordinate axes and the origin of coordinates is located in centre point of the upper surface, the z axis is directed downward vertically. Let U and W denote displacements along radius r and z directions, respectively, the physical equation of the circular plate can be shown as follows[3]

\[
\begin{align*}
\frac{\partial^4 U}{\partial r^4} + \frac{1}{r} \frac{\partial^3 U}{\partial r^3} + \frac{C_{11}}{C_{33}} \frac{1}{r} \frac{\partial^2 U}{\partial r^2} - C_{44} \frac{\partial W}{\partial z} &= 0, \\
\frac{\partial^4 W}{\partial r^4} + \frac{1}{r} \frac{\partial^3 W}{\partial r^3} + \frac{1}{r} \frac{\partial^2 W}{\partial r^2} + \frac{C_{11}}{C_{33}} \frac{\partial^2 U}{\partial r^2} - \frac{C_{44}}{C_{33}} \frac{\partial^2 U}{\partial z^2} &= 0,
\end{align*}
\]

Let \( a = \frac{\partial}{\partial r}, \quad C_1 = -\frac{C_{13}}{C_{33}}, \quad C_2 = C_{11} - \frac{C_{13}^2}{C_{33}}, \quad C_3 = C_{12} - \frac{C_{13} C_{23}}{C_{33}}, \quad C_4 = \frac{1}{C_{33}}, \quad C_5 = \frac{1}{C_{44}}, \quad R = r, Z = a. \) Eliminating \( \sigma, \) and \( \sigma_0 \) from Eq. (1), we obtain

\[
\begin{align*}
\begin{bmatrix}
\sigma_r \\
\sigma_\theta \\
\sigma_z \\
\tau_{rr} \\
\tau_{r\theta} \\
\tau_{\theta\theta} \\
\tau_{zz}
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 \\
C_{12} & C_{11} & C_{13} & 0 \\
C_{13} & C_{13} & C_{33} & 0 \\
0 & 0 & 0 & C_{44}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial U}{\partial r} \\
U/r \\
\frac{\partial W}{\partial z} \\
\frac{\partial W}{\partial r} + \frac{\partial U}{\partial z}
\end{bmatrix}.
\end{align*}
\]

Substitution of Eq. (2) into Eq. (1) and equilibrium equations of the plate gives the following state equation with a partial differentiation formula

\[
\frac{\partial}{\partial z} \begin{bmatrix}
U \\
W \\
R \\
Z
\end{bmatrix} =
\begin{bmatrix}
0 & -a & C_3 & 0 \\
C_1(a + 1/r) & 0 & 0 & C_4 \\
-C_2 a(a + 1/r) & 0 & 0 & C_1 a \\
0 & 0 & -(a + 1/r) & 0
\end{bmatrix}
\begin{bmatrix}
U \\
W \\
R \\
Z
\end{bmatrix}.
\]

Let

\[
U = \sum_n U_n(z)J_1(\xi_m r) + f(r)\bar{U}(z), \quad W = \sum_n W_n(z)J_0(\xi_m r),
\]

\[
R = \sum_n R_n(z)J_1(\xi_m r), \quad Z = \sum_n Z_n(z)J_0(\xi_m r).
\]

In which \( \bar{U}(z) \) is an unknown function for \( z \) and \( f(r) \) a given function for \( r \) which satisfies \( f(0) = 0, \xi_m = K_m/b, \) \( b \) is radius of the circular plate and \( K_m (m = 1, 2, \ldots) \) are the zero points of zeroth order Bessel function. It can be seen from Eq. (4) that the condition for displacement \( U \) equals zero in the plate centre under axisymmetric bending state has been satisfied. In the boundary conditions for simply supported or clamped circular plate, Eq. (4) satisfies the condition that the deflection \( W \) equals zero on the plate boundary, and the following boundary condition remains

1) Clamped circular plate, \( U |_{r = b} = 0, \) according to Eq. (4) yields

\[
\sum_n U_n(z)J_1(K_m) + f(b)\bar{U}(z) = 0;
\]

2) Simply supported circular plate, \( \sigma, |_{r = b} = 0, \) substituting Eq. (4) into Eq. (2) and