CATEGORICAL SYSTEM THEORY

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This is an investigation of natural systems from the standpoint of the mathematical theory of categories. It examines the relationships which exist between different descriptions through measurement of observables and dynamical interactions. We begin with a category theory of formal systems with observables, and then proceed to a category theory of dynamical systems. The two categories are then combined to represent natural systems. Topological considerations enter in the study of stability and bifurcation phenomena. Special emphasis is placed on natural systems which model biological processes. The categorical system theory developed is applied to the analysis of several biological problems and biological system theories.

1. Introduction. In his book The Scientific Outlook (1931), Russell described the 'scientific process' as composed of

"three main stages; the first consists in observing the significant facts; the second in arriving at a hypothesis which, if it is true, would account for these facts; the third in deducing from this hypothesis further consequences which can be tested by observation."

So according to Russell, the act of observation is basic to science. Intuitively, the notion of observable is attached to that of a concrete procedure for determining the value assumed by the observable of a system at a specific time. The crucial ingredient of any such procedure is a measuring instrument, which forms the basis both for our knowledge of the physical world and for our formulation of models which organize this knowledge and allow us to predict and control.

Rosen (1978), the single greatest influence in the development of this work, provides a comprehensive theory of observables and the descriptions arising from them. The theory is then applicable to any situation in which objects of interest are labelled by definite mapping processes, measurement in physics, pattern recognition, discrimination or classification. All of these diverse situations share a common character: namely, the generation of numbers (or other kinds of invariants) which serve to label the processes with which they are associated, such that processes are considered 'the same' if and only if they bear the same label. This leads to the idea of observable-induced equivalence relations (Section 3).
It is suggested in Rosen (1978) that a formal treatment of systems with observables using category theory would be a fruitful undertaking. The engagement in this problem marked the beginning of my dissertation. But why stop at systems with observables? Since the process of observation ultimately rests on the capacity of a given system to induce a dynamics (i.e. a change of state) in a measuring instrument (alias meter, recognizer, discriminator, classifier, etc.), it seems natural to consider systems with dynamics as well.

There is a reciprocity that exists in general dynamical interactions between systems. The process of measurement can be considered as a reciprocal induction of dynamics in both the system being measured and the system which measures. Then the basic problem in the analysis becomes this: to determine the observables through which a particular given dynamics is taking place, to specify the subsystems to which these observables belong and to identify the manner in which each of these subsystems is causing the others to change states.

The concepts of linkage of observables, stability and bifurcation, and their connections with dynamics are also treated in this paper in the context of the category of natural systems, an amalgamation of states, observables and dynamics. The categories constructed have curious links with diverse branches of mathematics from topology to Galois theory, and provide a natural setting to discuss the modelling relation. There are also many biological implications of categorical system theory. Among these, cellular dynamics, growth and aging and Rashevsky's (1972) organismic sets are given as examples. Other examples will appear in subsequent papers.

Category theory is the mathematical tool used in this work. The basic definitions, treated in any one of the standard texts on the subject (e.g. Mac Lane, 1971), are assumed.

2. Propositions. Throughout this study we will be dealing with three basic concepts: system, state and observable. Intuitively, a system is some part of the real world which is our object to study; a state is a specification of what our system is like at a particular time; and an observable of the system is some characteristic of the system which can, at least in principle, be measured. In other words, an observable of a system is a quantity which can induce dynamics in some appropriate meter.

These three basic concepts are interrelated via two fundamental propositions which we shall take as axioms in all of what follows:

**Proposition 1.** The only meaningful physical events which occur in the world are those represented by the evaluation of observables on states.