CONTROL OF THE LORENZ CHAOS BY THE EXACT LINEARIZATION

Chen Liqun (陈立群)  Liu Yanzhu (刘廷柱)

(Received May 13, 1996, Revised May 21, 1997; Communicated by Liu Zengrong)

Abstract

Controlling chaos in the Lorenz system with a controllable Rayleigh number is investigated by the state space exact linearization method. Based on proving the exact linearizability, the nonlinear feedback is utilized to design the transformation changing the original chaotic system into a linear controllable one so that the control is realized. Numerical examples of control are presented.

Key words  Rayleigh number, Lorenz system, chaos

I. Introduction

Recently controlling chaos has drawn much attention from the communities of physics, mechanics and engineering. Most of the control approaches were originally proposed by experimental or theoretical physicists and mathematicians, and no conventional control engineering strategies are employed. Here the state space exact linearization method in the nonlinear control system design is utilized to control chaos, and a typical chaotic dynamical system, the Lorenz system, treated as an example. The research concerning control of chaos in the Lorenz system is surveyed at first. Then the state space exact linearization method is presented for convenience. Afterwards the method is applied to design the nonlinear feedback control law to control the Lorenz chaos. Finally numerical examples are given.

II. Chaos in the Lorenz System and Its Control

The Lorenz system is a first dissipative model with chaotic behavior discovered in numerical experiment. Its state equations are

\[
\begin{align*}
    \dot{x}_1 &= P(x_2 - x_1) \\
    \dot{x}_2 &= R x_1 - x_2 - x_1 x_3 \\
    \dot{x}_3 &= x_1 x_2 - b x_3
\end{align*}
\]

A simple physical realization of system (2.1) is fluid loop thermosiphon heated form below and cooled form above. In this case, \( x_1 \) is the fluid velocity in the loop, \( x_2 \) and \( x_3 \) are the horizontal and vertical temperature difference respectively, \( P \) is analogous to the Prandtl number, \( b \) is a

* Project supported by the Science Foundation of the National Education Committee for Doctorate Program and the Applied Science Foundation of the Ministry of Metallurgical Industry

1Department of Engineering Mechanics, Shanghai Jiaotong University, Shanghai 200030, P. R. China
spatial constant, and $R$ is analogous to the Rayleigh number. Chaotic behavior of system (2.1) has been systematically studied\textsuperscript{19}. As controlling chaos has been paid much attention, there is some work on control of chaos in system (2.1).

With regard to control mechanism, the simplest one is the open-loop entrainment control. Based on the goal dynamical behavior, the external excitations $u_1(t), u_2(t), u_3(t)$ are designed so that the system

$$\begin{align*}
  \dot{x}_1 &= P(x_2 - x_1) + u_1(t) \\
  \dot{x}_2 &= Rx_1 - x_2 - x_1x_3 + u_2(t) \\
  \dot{x}_3 &= x_1x_2 - bx_3 + u_3(t)
\end{align*}$$

(2.2)

behavior asymptotically approaches the goal\textsuperscript{10-12}. The control can also be acted as parametric excitations\textsuperscript{13} called the parametric entrainment control

$$\begin{align*}
  \dot{x}_1 &= (P + u_1(t))(x_2 - x_1) \\
  \dot{x}_2 &= (R + u_2(t))x_1 - x_2 - x_1x_3 \\
  \dot{x}_3 &= x_1x_2 - (b + u_3(t))x_3
\end{align*}$$

(2.3)

The (parametric) entrainment control has some strict restrictions on the control goals and initial conditions. An open-plus-closed-loop control is applied in the Lorenz system by adding a closed-loop part in control $u_i(t)(i=1,2,3)$ for system (2.2)\textsuperscript{14}. Similarly, the parametric open-plus-closed-loop control\textsuperscript{15} can also be used in controlled system (2.3).

Feedback is introduced to increase control effectiveness and to reduce the number of control parameters. The most often studied case is

$$\begin{align*}
  \dot{x}_1 &= P(x_2 - x_1) \\
  \dot{x}_2 &= Rx_1 - x_2 - x_1x_3 + u(x_1,x_2,x_3) \\
  \dot{x}_3 &= x_1x_2 - bx_3
\end{align*}$$

(2.4)

Some approaches such as a linear feedback control and a bounded bang-bang control\textsuperscript{16}, conventional feedback control\textsuperscript{17}, nonlinear regulation based on feedback linearization\textsuperscript{18}, feedback control with global stabilization\textsuperscript{19} are adopted.

From the viewpoint of the actual physics, the Rayleigh number $R$ is easily controlled. For example, it can be varied by the heat applied to the bottom half of the fluid loop. Thus the controlled system

$$\begin{align*}
  \dot{x}_1 &= P(x_2 - x_1) \\
  \dot{x}_2 &= (R + u(x_1,x_2,x_3))x_1 - x_2 - x_1x_3 \\
  \dot{x}_3 &= x_1x_2 - bx_3
\end{align*}$$

(2.5)

is more easily realized physically than system (2.4). System (2.5) can be controlled by parameter variation and control duration\textsuperscript{20}, also be controlled by iteration sequences and interpolation for the corresponding Poincare map\textsuperscript{21}. However, with consideration for control engineering, there is little research on system (2.5). The only work is the proportional control and the proportional-plus-integral control based on the transfer functions of linearized system\textsuperscript{22}. Here we are going to design nonlinear feedback control law to control system (2.5).

There is other work on controlling chaos in Lorenz system, such as stochastic optimal