A DISCRETE ALGORITHM FOR COMPLEX FREQUENCY-DOMAIN CONVOLUTIONS *

Cai Kunbao (蔡坤宝)\(^1\), Yang Ruifang (杨瑞芳)\(^2\), Yu Jihui (俞黄辉)\(^1\)

(1. College of Electrical Engineering, Chongqing University, Chongqing 400044, P R China;
2. Biological Engineering Center, Chongqing University, Chongqing 400044, P R China)

(Communicated by Zhang Ruqing)

Abstract: A discrete algorithm suitable for the computation of complex frequency-domain convolution on computers was derived. The Durbin’s numerical inversion of Laplace transforms can be used to figure out the time-domain digital solution of the result of complex frequency-domain convolutions. Compared with the digital solutions and corresponding analytical solutions, it is shown that the digital solutions have high precision.

Key words: complex frequency-domain; convolution; Laplace transforms; numerical inversion

CLC numbers: O177.6; O174.5 Document code: A

Introduction

It is well-known that the Laplace transform of the product of two time-domain signals equals the complex frequency-domain convolution of individual Laplace transforms of two signals. Generally, the convolution of two complex frequency-domain analytical functions is calculated by using Cauchy’s residue theorem. It is necessary to find all of the poles of one function out of two. In engineering calculations, generally, it is difficult to get an analytical solution of most complex frequency-domain convolutions for the complexity of engineering problems. Moreover, it is often to give out the discrete forms of the Laplace transforms of time-domain signals. Thus, it is valuable to research into the algorithms for implementing the complex frequency-domain convolutions on computers.

1 Discrete Algorithm for Complex Frequency-domain Convolutions

With the complex frequency-domain convolution theory, the discrete algorithm of the convolutions suitable to implementing on computers is derived as follows.

1.1 Complex frequency-domain convolution theorem

If \(f(t)\) and \(g(t)\) are causal signals and

---

* Received date: 1998-05-08; Revised date: 2000-01-20

Foundation item: the National Natural Science Foundation of China (39470147)

Biography: Cai Kunbao (1950 ~ ), Doctor, Associate Professor
Cai Kunbao, Yang Ruifang and Yu Jihui

\[ L[f(t)] = F(s), \quad \text{Re}[s] > \sigma_f, \]
\[ L[g(t)] = G(s), \quad \text{Re}[s] > \sigma_g, \] 

then

\[ F_w(s) = L[f(t)g(t)] = \frac{1}{2\pi j} \int_{-\infty}^{\infty} (s) G(s - z) dz. \] 

The integral is taken along a straight-line \( c \) parallel to the imaginary axis in the complex \( z \)-plane with

\[ \sigma = \text{Re}[s] > \sigma_f + \sigma_g, \quad \sigma_f < c < \sigma - \sigma_g. \] 

With this choice, the all poles of function \( F(z) \) must lie to the left of integral path \( c \) and the all poles of function \( G(s - z) \) must lie to the right of integral path \( c \). That is the so-called complex frequency-domain convolution theorem [13].

1.2 Discrete algorithm

Substituting \( z = c + j\omega' \) into Eq. (2), we get

\[ F_w(s) = \frac{1}{2\pi j} \int_{-\infty}^{\infty} F(c + j\omega') G(s - (c + j\omega')) d\omega'. \] 

Substituting \( \omega' = 2\pi n / T \) into Eq. (4), we have

\[ F_w(s) = \frac{1}{T} \sum_{n=-\infty}^{\infty} F(c + j2\pi n / T) G(s - (c + j2\pi n / T)). \] 

Discretizing the variable \( s \) into

\[ s_k = a + jk2\pi / T, \quad a = 2c, \] 

we get

\[ F_w(s_k) = \frac{1}{T} \sum_{n=-\infty}^{\infty} F(c + j2\pi n / T) G(c + j2\pi (k - n) / T). \] 

Under a permissible error tolerance

\[ F_w(s_k) \approx \frac{1}{T} \sum_{n=-M+1}^{M-1} F(n) G(k - n) = \frac{1}{T} F(k) \ast G(k), \] 

where

\[ F(k) = F(c + j2\pi k / T), \quad G(k) = G(c + j2\pi k / T), \] 

The linear convolution represented by Eq. (8) can be implemented by using FFT to perform circular convolution. We know that the final computational result in the complex frequency-domain will return to the time-domain. It does not matter whether the result of the complex frequency-domain convolution continues to take part in other computations in the complex frequency-domain or immediately returns to the time-domain. Therefore the selection of three parameters, \( T, a \) and \( M \) in above formulae should follow the convergence criterion given by the Durbin’s numerical inversion of Laplace transforms [2]. Particularly, the selection of parameter \( M \) should satisfy that the value of \( | F_w(s_M) | \) can be ignored under a given error tolerance. If the