FINITE ELEMENT ANALYSIS OF WAVE PROPAGATION
IN FLUID-SATURATED POROUS MEDIA

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Abstract: With the porous media model based on mixture theory, a finite element formulation for dynamic transient analysis of fluid-saturated two-phase porous media is presented. Time integration of the equation, deduced with penalty method, can be performed by using implicit or explicit method. One-dimensional wave propagation in column under step loading and impulsive loading are analyzed with the developed finite element program. The obtained curves of displacements, velocities, effective stresses and pore pressures against time demonstrate the existence of wave propagation phenomena, which coincide with the theoretical results.

Key words: porous media; wave propagation; finite element method

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Introduction

The dynamic transient response analysis of porous media plays a very important role in a lot of engineering practices such as transient consolidation, noise control, earthquake engineering and bioengineering. Biot[1] originally discussed the wave propagation problem in fluid-saturated porous media, whose theory and results were extensively accepted and become the standard reference of the following porous media models developed by other authors. Zienkiewicz and Simon et al.[2,3], based on Biot's model, comprehensively and deeply researched the dynamic transient problem with finite element method. Although Biot's model were broadly accepted and applied, it has some shortcomings because it originated from experience, lacking strong mechanic foundation.

Establishment of mixture theory based on continuum mechanics resulted in the modern porous media theory, which is viewed as a mixture theory restricted by volume fraction[4]. In the framework of modern continuum mechanics, Bowen[5,6] proposed incompressible and compressible porous media models, with volume fraction as an independent variable. This type of models are attracting more and more researchers because they are constructed from continuum mechanics theory and can be easily extended and developed[4]. With porous media model based on mixture theory, literature [7] obtained analytical result, using Laplace transformation method, of one-dimensional wave propagation in fluid-saturated porous media and finally gave out the concrete dynamic responses of one-dimensional column under both step loading and impulsive loading, which are good reference of the thereafter available numerical methods.

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In this paper, Bowen's incompressible porous media model\cite{a1} was adopted, and a dynamic finite element formulation of fluid-saturated two-phase porous media was deduced with penalty method. Both implicit and explicit time integration methods can be used to solve the dynamic equation. The dynamic transient responses of one-dimensional column under both loading cases, analyzed with the developed finite element program, are obtained.

1 Governing Equations

The porous media of mixture theory is an immiscible mixture consisting of multiple constituents, each of which is continuum medium with independent motion. The geometrical and physical variables of each constituent are defined in entire space. The mixture as a whole satisfies all balance equations which a single phase medium meets. Volume fraction is an important concept in this theory, it is defined as the ratio of volume of a constituent to that of the whole mixture.

The local mass balance equation of a constituent $\phi^a$ can be expressed as

$$\left( \rho^a \right)_t + \rho^a \text{div} x^a = \hat{\rho}^a,$$

(1)

here $x^a$ is the velocity of a constituent, the notation of $(\cdots)_t$ stands for material time derivative, $\rho^a = n^a \rho^R$ is the macroscopic mass density, $\rho^R$ the microscopic mass density, and $n^a$ the volume fraction. $\hat{\rho}^a = \hat{\rho}^a(x, t)$ represents the mass supply of all other constituents to constituent $\phi^a$ occupying position $x$ at time $t$. From the local mass balance definition of mixture, here is

$$\sum_{a=1}^{k} \hat{\rho}^a = 0,$$

(2)

i.e. the sum of mass supply of all $k$ constituents vanishes.

The local linear momentum balance of constituent $\phi^a$ is

$$\text{div} T^a + \rho^a (b^a - x^a) + \hat{p}^a = 0,$$

(3)

where $x^a$ denotes the acceleration of the particles of each constituent. $T^a$ is the partial Cauchy stress tensor, $b^a$ the external acceleration. $\hat{p}^a = \hat{p}^a(x, t)$ indicates linear momentum supply per unit volume. According to the linear momentum balance of mixture,

$$\sum_{a=1}^{k} (\hat{p}^a + \rho^a x^a) = 0$$

(4)

is obtained.

The local balance equation of moment of momentum of each constituent $\phi^a$ gives out

$$T^a = T^{aT} - \hat{M}^a,$$

(5)

here $\hat{M}^a = \hat{M}^a(x, t)$ is a coupled skew symmetric tensor of moment of momentum, which is concerned with local moment supply of momentum. The balance idiom of moment supply of momentum results in

$$\sum_{a=1}^{k} \hat{M}^a = 0.$$  

(6)

The fluid-saturated two-phase porous medium discussed here is assumed to be chemically inert, and heat as well as the exchanges of moment of momentum between solid and liquid are excluded. The balance equations of mass and momentum are given as follows;