THE VARIATIONAL PRINCIPLES AND APPLICATION OF NONLINEAR NUMERICAL MANIFOLD METHOD *

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Abstract: The physical-cover-oriented variational principle of nonlinear numerical manifold method (NNMM) for the analysis of plastic problems is put forward according to the displacement model and the characters of numerical manifold method (NMM). The theoretical calculating formulations and the controlling equation of NNMM are derived. As an example, the plate with a hole in the center is calculated and the results show that the solution precision and efficiency of NNMM are agreeable.

Key words: variational principle; numerical manifold method; nonlinear analysis; plastic flow

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Introduction

The finite cover techniques, similar to the conception of finite cover used in manifold analysis of modern mathematics, are introduced into Numerical Manifold method (NMM) and the finite covers consist of mathematical covers and physical covers which can be separated. The NMM is considered as an all-new and prosperous numerical analysis method that can deal with continuous and discontinuous mechanical problems uniformly. But up to now, the theoretical system and applications of NMM is confined to the linear analysis [1] or can only deal with some special problems [2]. In engineering practices it is nonlinear (geometry or material nonlinear) problems that often occur. In order to make NMM to be widely used in engineering fields, it is urgent to set up the theoretical system of Nonlinear Numerical Manifold Method (NNMM). In this paper, the physical-cover-oriented variational principles of Nonlinear Numerical Manifold Method (NNMM) for the analysis of nonlinear problems is put forward according to the displacement model and the characteristics of Numerical Manifold Method. The theoretical

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calculating formulations and the controlling equations of NNMM are derived. As an example, the numerical analysis results are shown. The calculating results show that NNMM is a high efficiency numerical analysis method with high solution precision. By further expanding, the NNMM can be widely used in practical engineering fields.

1 The Basic Equations of Plastic Flow Theory and Variational Principles

With Euler description, continuous body is in equilibrium under static condition at time t. Suppose the stress state and loading history are known. The external force increments are \( \text{d}p_i \) on stress boundary \( \Gamma_p \) and displacement increments are \( \text{d}u_i \) on displacement boundary \( \Gamma_u \). All the equations are linear when these increments are infinitesimal. The basic equations are as follows:

1) Equilibrium Equations:
\[
\text{d}\sigma_{y,j} = 0; \tag{1}
\]

2) Strain-Stress Relations:
\[
\text{d}e_{y} = \frac{1}{2}(\text{d}u_{i,j} + \text{d}u_{j,i}); \tag{2}
\]

3) The relation of stress increments \( \text{d}\sigma_{y} \) and strain increments \( \text{d}e_{y} \) is linear;

4) Stress Boundary Conditions:
\[
\text{d}\sigma_{y}n_j = \text{d}\bar{p}_i \quad (\text{in } \Gamma_p); \tag{3}
\]

5) Displacement Boundary Conditions:
\[
\text{d}u_{i} = \text{d}\bar{u}_i \quad (\text{in } \Gamma_u). \tag{4}
\]

The principle of minimum potential energy of plastic flow theory can be described as\(^5\):

With all \( \text{d}u_{i} \) and \( \text{d}e_{y} \) which are smooth enough and satisfy Eqs. (2) and (4), the \( \text{d}u_{i} \) which makes the following functional
\[
\Pi_{dp} = \int_{\Omega} [A(\text{d}e_{y}) - dF_{i} \text{d}u_{i}]d\Omega - \int_{\Gamma_p} \text{d}\bar{p}_i \text{d}u_{i} d\Gamma \tag{5}
\]
minimum must be the exact solution of displacement increments.

For stress-hardening materials, the strain-stress relation is
\[
\text{d}e_{y} = \frac{(1 - 2\nu)}{3E} \text{d}\sigma_{kk} \delta_{y} + \frac{\text{d}\sigma_{y}^{p}}{2G} + a^{*} H \frac{\partial f}{\partial \sigma_{y}} \text{d}f,
\]
increment strain density function \( A(\text{d}e_{y}) \) is
\[
A(\text{d}e_{y}) = \frac{E}{(1 - 2\nu)} \text{d}\sigma_{kk} \text{d}e_{y} + G \text{d}\sigma_{ik} \text{d}e_{ik} - a^* \frac{G}{1 + \nu} \frac{\partial f_{\sigma_{ma}}}{\partial \sigma_{y}} - a^* \frac{\partial f_{\sigma_{ma}}}{\partial \sigma_{y}} - \frac{\partial f}{\partial \sigma_{y}} \text{d}f. \tag{7}
\]

For ideal plastic materials, the strain-stress relation is\(^4\)
\[
\text{d}e_{y} = \frac{(1 - 2\nu)}{3E} \text{d}\sigma_{kk} \delta_{y} + \frac{\text{d}\sigma_{y}^{p}}{2G} + a^* H \frac{\partial f}{\partial \sigma_{y}} \text{d}f,
\]
\[
\lim_{\nu \to 0, \sigma \to 0} \frac{\partial f}{\partial \sigma_{y}} = \text{d}l > 0. \tag{9}
\]
increment strain density function \( A(\text{d}e_{y}) \) is