EXTENDED SELF-SIMILAR SCALING LAW OF MULTI-SCALE EDDY STRUCTURE IN WALL TURBULENCE *

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Abstract: The longitudinal fluctuating velocity of a turbulent boundary layer was measured in a water channel at a moderate Reynolds number. The extended self-similar scaling law of structure function proposed by Benzi was verified. The longitudinal fluctuating velocity in the turbulent boundary layer was decomposed into many multi-scale eddy structures by wavelet transform. The extended self-similar scaling law of structure function for each scale eddy velocity was investigated. The conclusions are 1) The statistical properties of turbulence could be self-similar not only at high Reynolds number, but also at moderate and low Reynolds number, and they could be characterized by the same set of scaling exponents $\xi_1(n) = n/3$ and $\xi_2(n) = n/3$ of the fully developed regime. 2) The range of scales where the extended self-similarity valid is much larger than the inertial range and extends far deep into the dissipation range with the same set of scaling exponents. 3) The extended self-similarity is applicable not only for homogeneous turbulence, but also for shear turbulence such as turbulent boundary layers.

Key words: wavelet transform; eddy; scaling law

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Introduction

Much work has been devoted in the last few decades to the measurement and modeling of the scaling law of structure function of turbulent flows. The so-called "velocity structure function of order n" for turbulent flows is defined as $\langle \Delta V(r)^n \rangle$, where $\Delta V(r) = V(x+r) - V(x)$ is the velocity component increment parallel to the relative displacement r of two positions separated by a distance of r in the flow field.

Let us remember that the research of structure function scaling law is usually limited by the following assumptions: 1) in the full developed turbulent flow so that the Reynolds number is infinite; 2) local homogeneous and isotropic; 3) for r in the inertial range.

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The expectation of the scaling law is that
\[ \langle AV(r)^n \rangle \propto r^\xi(n) \quad (\eta \ll r \ll L), \] (1)
where \( \eta \) is the dissipate length, \( L \) is the integral scale and \( \xi(n) \) is called scaling exponent. The scaling law is an indication of the existence of scale invariance in turbulence.

For the third-order structure function, one can deduce the following Kolmogorov relation within above assumptions from the Navier-Stokes equations:
\[ \frac{4}{5} \frac{d}{dr} \langle AV(r)^3 \rangle = -4 \nu \frac{d^2 \langle AV(r)^2 \rangle}{dr^2}, \] (2)
where \( V \) is the kinematics viscosity, \( \langle \rangle \) stands for ensemble averaging and \( \epsilon \) is the average rate of energy dissipation per unit mass. Within the inertial range, the second term on the right-hand side in Eq. (2) can be neglected.

\[ \langle AV(r)^3 \rangle = -\frac{4}{5} \epsilon r. \] (3)

This means that: \( \xi(3) = 1 \).

The classical Kolmogorov theory predicts that: \( \xi(n) = \frac{n}{3} \).

Benzi et al. \cite{11} recently showed evidence for the so-called extended self-similarity in their measurements of turbulence generated either by a wake flow past a cylinder or by a jet at moderate Reynolds number:
\[ \langle AV(r)^n \rangle = A_n \langle AV(r)^3 \rangle \xi(n) = B_n \langle AV(r)^3 \rangle \xi(3), \] (6)
where \( A_n \) and \( B_n \) are two different sets of constant independent of \( r \). Since the third-order structure function \( \langle AV(r)^3 \rangle \) is proportional to \( r \), instead of plotting the \( n \)th-order structure function \( \langle AV(r)^n \rangle \) against \( r \), they plotted the \( n \)th-order structure function \( \langle AV(r)^n \rangle \) against the third-order structure function \( \langle AV(r)^3 \rangle \). Eq. (6) is valid not only in the fully developed turbulence but also at moderate and low Reynolds number, even if no inertial range is established. Moreover, it has been shown that the range of scales where Eq. (6) valid is much larger than the inertial range and extends far deep into the dissipation range.

Stolovitzky (1993)\cite{12} repeated the experiments of Benzi and presented their experimental results. They measured the time series of the fluctuating velocity at a moderate Reynolds number in a turbulent boundary layer over a flat plate and investigated the extended self-similarity of the structure function. They revealed that, for low-order moments, a single scaling law with the same scaling exponents not far from \( \xi(n) = n/3 \) for dissipate as well as inertial range. However, as the order of the moment increases, the scaling law in the dissipate region and in the inertial region separates out. Within the dissipate region, the scaling exponents are nearly given by \( \xi(n) = n/3 \) with \( \xi(8) = 2.66 \), \( \xi(8) = 2.42 \). For \( r \) in the inertial range, the plot of \( \xi \) versus consists of another linear region of slope \( \xi(8) = 2.05 \), \( \xi(8) = 2.12 \) slightly less than \( \xi(n) = n/3 \), joined by a smooth transitional region. The difference between the two regimes becomes increasingly apparent for higher \( n \).

1 Experimental Apparatus and Techniques

Experiments were conducted in a full-developed turbulent flow of a free-surface water channel. Velocity measurements in the water channel were taken by TSI anemometer system with