ENERGY AND BIOLOGICAL EVOLUTION—I.
THE EQUILIBRIUM STATES OF BIOCHEMICAL PROCESSES*

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The Thom gradient model of morphogenesis poses the following *a posteriori* problem: "From the observed morphology of a given natural process (effect) determine the dynamics of the process (cause)". In this paper we consider the classical *a priori* problem: "Given the cause (dynamics) determine the effect (resultant morphology)".

We find that in biochemical processes the mechanisms for energy activation, energy-matter interaction and energy dissipation determine the dynamics. Furthermore there exists basic energy mechanisms which drive the equilibrium states through the elementary catastrophes of Thom. A comparison with current theories shows that our models describe open ecological food chains and their dynamical systems generalize the equations of organisation posed by M. Eigen.

Introduction. In this paper (Part I) we present biochemical reaction schemes whose energy structures drive the equilibrium states from the Morse (Logistics Equation) via the fold successively to the umbilics. Part II is devoted to an analysis of the mathematical structure revealed by the models. In Part III we discuss applications in theoretical ecology and the self-organisation of biological macromolecules.

A. From the Morse to the Cusp.

1. A generic growth law. Let $\rho$ denote the density of a biological organism whose growth depends on an energy species of density $K$. For the moment we suppose that the growth and death rates of $\rho$ (denoted by $r$ and $d$) are constant. We wish to examine the evolution of $\rho$ subject to each of the following assumed mechanisms of energy.

   $E1$. $K$ is constant and $\rho$ grows via continuous activation by $K$ in accordance with the scheme,

   \[ \frac{d\rho}{dt} = r(K) \]

   \[ (K) \]

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E2. $K$ is not constant and scheme (1) is valid.

The following results are evident.

R1. Given E1, the growth law for $\rho$ is Malthusian (exponential)

$$\frac{d\rho}{dt} = (rK - d)\rho.$$

R2. Given E2, the growth law for $\rho$ is logistic and $\rho$ and $K$ satisfy an 'energy' conservation law

$$K + \rho = \text{constant} = K_0,$$

where $K_0$ is the total energy of the environment interpreted as the value of $K$ when $\rho = 0$.

The mechanism E2 yields the dynamical system

$$\frac{dK}{dt} = -rK\rho + \frac{d\rho}{dt} \quad (1.1)$$
$$\frac{d\rho}{dt} = rK\rho - \frac{d\rho}{dt}. \quad (1.2)$$

Addition of these equations gives the conservation law

$$K + \rho = K_0. \quad (1.3)$$

Substituting $K = K_0 - \rho$ in (1.2) yields the logistics growth law

$$\frac{d\rho}{dt} = r(E - \rho)\rho, \quad E = K_0 - \frac{d}{r}. \quad (1.4)$$

The evolution equation (1.4) is equivalent to the abstract reaction scheme (1.5):

$$E + \rho \rightleftharpoons 2\rho, \quad (1.5)$$

where $E$ is a constant energy species and $r \neq 0$.

Remarks. Without further activation by the environment the evolution of $\rho$ must satisfy the conservation law (1.3) with $K_0$ the maximum supporting capacity of the environment. If $\rho$ is to evolve to higher equilibrium configurations additional energy must be injected into the system. Equivalently, equation (1.4) must be perturbed. The nature of such a perturbation will determine the future morphology of $\rho$. This is the crux of the whole paper. We present biochemical models of additional energy mechanisms which drive the equilibrium configurations of $\rho$.