THE $\dot{V}_A/Q$ RESOLUTION OF INERT GAS DATA

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The non-uniqueness of $\dot{V}_A/Q$ distributions satisfying inert gas retention data without error is studied. The ability of such data to resolve blood flows at particular $\dot{V}_A/Q$ values is discussed through the application of linear programming and Backus–Gilbert theory. It is shown that the resolution deteriorates away from the extremes of low and high $\dot{V}_A/Q$.

1. Introduction. Since the inequality of ventilation and blood flow in a lung is responsible for most of the defective gas exchange in pulmonary disease, one of the goals of the respiratory physiologist is to draw the frequency distribution of ventilation–perfusion ratios within the abnormal lung. In recent years, the use of non-respiratory gases as pulmonary indicators has been appreciated, and it has been repeatedly suggested that measurement of multiple inert gas elimination would offer the most information about the comprehensive double distribution of ventilation and blood flow over their ratio. These suggestions appeared to culminate when Wagner et al. (1974a) reported that virtually continuous $\dot{V}_A/Q$ distributions could be defined using only a few inert gas retention measurements.

When the amount of an inert gas in the alveolar gas volume is maintained constant by its continuous intravenous infusion, its relative arterial retention was shown by Farhi (1967) to be simply related to the blood flow distribution over $\dot{V}_A/Q$ by

$$R_G = \frac{P_{aG}}{P_{vG}} = \int_0^\infty \frac{\lambda_GQ(\dot{V}_A/Q)}{\lambda_G + \dot{V}_A/Q} \, d(\dot{V}_A/Q),$$

(1)

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where \( Q(V_A/Q) \) is the normalized blood flow per unit \( V_A/Q \), and \( \lambda \) is the Ostwald blood:gas partition coefficient, herein simply referred to as the "solubility", of the inert gas \( G \). The relative excretion of the gas is analogously related to the normalized ventilation, and since the mathematics is identical, the distribution of ventilation will not be discussed.

Farhi realized that inert gas elimination would be a useful tool for assessing \( V_A/Q \) inhomogeneity. He noted that such a technique should detect and group spatially scattered lung units with any particular ventilation-perfusion ratio. This would then complement radio isotope scanning techniques which reveal the topography of averaged relationships but cannot detect flows in a particular ratio unless they occupy a definite volume.

Should the continuous functional dependence of retention upon solubility be known, e.g. by curve-fitting the data, then one can formally invert (1) to obtain the blood flow distribution. Setting \( x = V_A/Q \):

\[
\frac{R(\lambda)}{\lambda} = \int_0^\infty Q(x) \left[ \int_0^\infty e^{-s\lambda} e^{-sx} \, ds \right] \, dx
\]

and it follows from the definition of the Laplace transform and the applicability of Fubini's theorem that

\[
\frac{R(\lambda)}{\lambda} = \mathcal{L} \left[ \mathcal{L}(Q(x)) \right],
\]

whence

\[
Q(x) = \mathcal{L}^{-1} \left[ \mathcal{L}^{-1} \left( \frac{R(\lambda)}{\lambda} \right) \right],
\]

which, in the sense of Lerch's lemma, is a unique solution.

The distribution of blood flow over \( V_A/Q \) is thought to be continuous because the causes of inhomogeneity are multifaceted and lung structures so complexly interrelated. Yet Wagner et al. (1974a) chose to consider discrete values for the variable \( V_A/Q \), and so compartmentalize the blood flow. With \( V_A/Q \) values equally spaced logarithmically, they reported blood flows in