ON STOCHASTIC MODELS OF LIGHT ABSORPTION

J. E. MANN
Department of Statistics,
Virginia Polytechnic Institute and State University,
Blacksburg, VA 24061, U.S.A.

Previously developed light absorption models have treated the effective quantity of light-absorbing material within the experimental environment as a constant, i.e. a parameter. These models are, however, probabilistic in nature and are properly applied and interpreted only in a statistical sense. Thus, it is clearly logical to regard the effective quantity of light-absorbing material to be a random variable. In this paper an asymptotic distribution is derived for this random quantity, and it is shown how this distribution may be incorporated into present models. These results may be applied to light absorption by plant and crop canopies as well as to liquid or solid media. Furthermore, previous models are based upon the assumption that light is parallel, or effectively so, as for solar light. Such models may be inadequate for an artificial (laboratory) environment which utilizes point source light. Present models for estimating light interception and radiation intensity are modified so as to accommodate a proximate point source of light. Numerical examples are included to illustrate the theory.

1. Introduction. Models of direct beam parallel light interception developed by Mann (1979) estimate the fraction of incident radiation intercepted by an absorbing medium and the mean radiation intensity beneath the medium. Various details of the models were first formulated in Mann (1977), Mann et al. (1977), Mann and Curry (1977) and Mann et al. (1979). These models treat the effective quantity of light-absorbing material within the region of application as if it were a parameter. The models are, however, properly interpreted only in a statistical sense. Therefore, the effective quantity of light-absorbing material is random and should be so regarded. In this paper an appropriate asymptotic distribution for this random quantity is derived. It is shown how this distribution is to be utilized along with present models.

Previous models are applicable only for the case of parallel light. For reviews of previous models see Liou (1980) and LeMeur and Blad (1974). Since an artificial environment may utilize point source light, existing models may be inadequate. Models useful for proximate point source light are developed. Examples are given to illustrate the use of the models.

2. Review. In this section essential notation is defined and previously developed light interception models which are to be utilized in this paper are listed for easy reference.
Suppose that a large number of small light-absorbing bodies are independently distributed within a transparent medium in accordance with a common, absolutely continuous probability law, which is assumed to be known. Let $R_0$ be a plane region, of area $A$, beneath the medium. A beam of parallel light rays is directed at $R_0$ so that a ray may intercept $R_0$ only by penetrating the medium.

Consider projecting the suspended absorbing bodies onto $R_0$ along lines parallel to the beam of light. Denote by $K$ the total area of these plane projections, and by $g(u, v)$ the probability density of the locations of these projections in $R_0$. Note that a projection may be located by its center of mass or by some other suitably chosen point.

It is recognized that $K$ is a random variable; however, in order to simplify the presentation of formulae we shall temporarily treat $K$ as if it were a known constant. In such cases, the results are conditional on $K$. If the probability distribution of $K$ is known, then unconditional formulae may be obtained by averaging the conditional results with respect to this distribution. An asymptotic distribution for $K$ will be derived in Section 3. The formulae which are required for the development of our models will now be listed. These may be found in Mann (1979).

Assuming that a light ray which encounters an absorbing body is totally absorbed (black bodies), the average fraction of $R_0$ which is in direct light is estimated by

$$E(L) = A^{-1} \int_{R_0} \exp \left[ -Kg(u, v) \right] d(u, v),$$

where the integral is a double integral over $R_0$.

On the other hand, if a ray is only partially absorbed and $t^*$ is the average effective transmission coefficient of the absorbing bodies over a spectral range, then the average radiation intensity over $R_0$ is approximated by

$$m(I) = IA^{-1} \int_{R_0} \exp \left[ -K(1-t^*)g(u, v) \right] d(u, v),$$

where $I$ is the radiation intensity (assumed to be uniform) incident upon the medium.

Before proceeding, several observations regarding formulae (1) and (2) should be made. These formulae measure direct light penetration only; scattering and reflection are ignored. The derivations of these formulae did not account for stochastic variation in the sizes of absorbing bodies. Disregarding such variation may be justified on the basis of analytical