ON THE EFFECT OF THE CONCENTRATION PROFILE OF RED CELLS ON BLOOD FLOW IN THE ARTERY WITH STENOSIS

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The effects of the viscosity-concentration dependence and of the concentration profile on blood flow through a vessel with stenosis have been studied. The flow resistance and the wall shear stress have been found to be smaller than in the two-fluid model with constant viscosities.

1. Introduction. The effects of peripheral layer viscosity on blood flow through an artery with stenosis have recently been studied (Shukla et al., 1980a). They assumed a two-fluid model in which both fluids are Newtonian with constant viscosities. The non-Newtonian behaviour of blood was considered later (Shukla et al., 1980b, c). In this paper we study the steady laminar flow of the suspensions of rigid red cells. We assume different concentration profiles and different viscosity-concentration dependences. The effects of the concentration profile on the flow resistance and on the shear stress are studied.

2. Theoretical Analysis. We suppose here that the stenosis is symmetrical (Shukla et al., 1980a; Young, 1968). The radius of the vessel is given by

$$R(z) = \begin{cases} 
1 - \frac{\delta_z}{2R(0)} \left[ 1 + \cos \frac{2\pi}{L_0} \left( z - d - \frac{L_0}{2} \right) \right], & z \in (d, d + L_0), \\
1, & z \in (d + L_0, d + L_0). 
\end{cases}$$

(1)
Here the $z$-axis is along the vessel axis, $L_0$ and $\delta_z$ are the length and the maximum height of the stenosis, and $R(0)$ is the nonconstricted radius of the vessel.

The resistance to flow $\lambda$ and the wall shear stress at the maximum height of the stenosis $\tau_s$ are, in the presence of stenosis (Shukla et al., 1980a),

\[
\lambda = \frac{2}{\pi} \left[ \frac{L-L_0}{I(0)} + \int_{d}^{d+L_0} \frac{dz}{I(z)} \right]
\]  

(2)

and

\[
\tau_s = \left[ \frac{R(z)\dot{V}}{\pi I(z)} \right]_{z=d+(L_0/2)}.
\]

(3)

In equations (2) and (3) $\dot{V}$ is the volume flow rate, $L$ is the length of the vessel and

\[
I(z) = \int_{0}^{R(z)} \frac{r^3}{\mu(r)} dr,
\]

(4)

where $\mu(r)$ is the dynamic viscosity function.

We consider a slow viscous flow of suspensions of rigid red cells. As a reasonable approximation for the viscosity function we assume

\[
\mu(\xi) = \mu_p \left[ 1 - k c_v(\xi) \right]^{-2.5},
\]

(5)

where $\mu_p$ is the viscosity of the plasma, $\xi = r/R(z)$, $k = 1.6$ and $c_v(\xi)$ is the volume concentration of the red cells (Chien et al., 1967; Brooks et al., 1970; Cokelet, 1972). Optimization of viscous energy dissipation yields also a similar expression for the viscosity as equation (5), but with slightly different values for the parameters (Quemada, 1977). Equation (5) yields the approximation

\[
\mu(\xi) \approx \mu_p \exp(4c_v(\xi))
\]

(6)

for dilute suspensions ($c_v(\xi) \ll 0$, 2).

The concentration of red cells is assumed to be a piecewise linear function given by

\[
c_v(\xi) = c \left[ \theta(\xi, \xi_\gamma - \xi) + \frac{\xi_\beta - \xi}{\xi_\beta - \xi_\gamma} \theta(\xi_\gamma - \xi) \theta(\xi - \xi_\gamma) \right]
\]

(7)