ON A NEW ALGORITHM OF CONSTRUCTING SOLITARY WAVE SOLUTIONS FOR SYSTEMS OF NONLINEAR EVOLUTION EQUATIONS IN MATHEMATICAL PHYSICS *

Yan Zhenya (闫振亚), Zhang Hongqing (张鸿庆)
(Institute of Mathematical Science, Dalian University of Technology, Dalian 116024, P.R. China)

Abstract: According to the improved sine-cosine method and Wu-elimination method, a new algorithm to construct solitary wave solutions for systems of nonlinear evolution equations is put forward. The algorithm has some conclusions which are better than what the hyperbolic function method known does and simpler in use. With the aid of MATHEMATICA, the algorithm can be carried out in computer.

Key words: system of nonlinear evolution equations; sine-cosine method; Wu-elimination method; solitary wave solution

CLC number: O175 Document code: A

Introduction

Solving nonlinear equations is always a very interesting subject for mathematician and physician, in particular, solitary wave solutions for nonlinear equations are of both theoretical and practical importance. Recently, Yan[1] obtained a transformation directly from the famous sine-Gordon equation, and then provided a so-called sine-cosine method used to solve nonlinear wave equation. But he only applied the method to a simple nonlinear equation, the method was not be applied to solve systems of nonlinear equations.

In this paper, we try to improve the sine-cosine method and apply it to construct solitary wave solutions for systems of nonlinear evolution equations, finally, more solitary wave solutions are obtained. With the aid of MATHEMATICA and Wu-elimination method, the algorithm can be carried out in computer.

The main ideas of the algorithm are as follows

For the system of nonlinear evolution equations given

\[ u_t - F(u, v, u_x, v_x, \cdots) = 0, \quad (1) \]
\[ v_t - G(u, v, u_x, v_x, \cdots) = 0. \quad (2) \]

By using the travelling wave transformation, setting (1) and (2) have the following form solution
where \( k, \lambda \) are constants to be determined later and \( c \) is arbitrary constant. \( \psi, \theta \) are functions w.r.t \( \alpha \) to be determined later.

Substituting (3) into (1) and (2), respectively, yields

\[
\begin{align*}
\psi' &= F_1(\psi, \theta, \phi', \theta', \cdots) = 0, \\
\theta' &= G_1(\psi, \theta, \phi', \theta', \cdots) = 0,
\end{align*}
\]

where \( \cdots \) denotes \( \frac{d}{d\alpha} \). Setting (4) and (5) have the following form solutions

\[
\begin{align*}
\psi(\alpha) &= \sum_{i=1}^{n} \cos^{i-1}\omega (A_i \cos \omega + B_i \sin \omega) + A_0, \\
\theta(\alpha) &= \sum_{i=1}^{n} \cos^{i-1}\omega (a_i \cos \omega + b_i \sin \omega) + a_0
\end{align*}
\]

and

\[
d\omega/\alpha = \sin \omega,
\]

where \( A_0, A_i, B_i (i = 1,2, \cdots, n) \) and \( a_0, a_j, b_j (j = 1,2, \cdots, m) \) are constants to be determined later.

**Step 1** Equating the highest order nonlinear term and the highest order linear partial derivative term in (1) and (2), respectively, yields the values of \( m, n \).

**Step 2** Substituting (6) – (8) into (4) and (5), respectively, yields two polynomial equations w.r.t \( \sin \omega \cos^p \omega, \cos^{p+1} \omega (p = 0,1,2, \cdots) \), with the aid of MATHEMATICA, the step can be carried out in computer.

**Step 3** Setting the constant term and coefficients of \( \sin \omega, \cos \omega, \sin \omega \cos \omega, \cos^2 \omega, \cdots \) in the two equations obtained in step 2 to zero, yields a system of algebraic equations w.r.t the unknown numbers \( k, \lambda, A_0, A_i, B_i (i = 1,2, \cdots, n), a_0, a_j, b_j (j = 1,2, \cdots, m) \).

**Step 4** Using Wu-elimination method\(^{[5,3]} \), the algebraic equations in step 3 are solved in computer, and then combining (3) ~ (8), yields the solitary wave solutions for systems (1) and (2).

As the applications of the algorithm, we consider the two examples, the first one is the system of Whitham-Broer approximate equations with long water waves in two dimensional spaces

\[
\begin{align*}
\psi_t + H_x + H_y + uu_x + uu_y + \sigma_1 u_{xx} + \sigma_2 u_{yy} &= 0, \\
\theta_t + Hu_x + Hu_y + uH_x + uH_y - \sigma_1 H_{xx} - \sigma_2 H_{yy} &= 0,
\end{align*}
\]

where \( \sigma_1, \sigma_2 \) are constants. The second one is the system of the coupled MKdV equations in two dimensional spaces.

1 2 + 1-Dimensional Solitary Wave Solutions for the Equations (9) and (10)

During the study of water wave, Whitham and Broer\(^{[4,5]} \) found a system of approximate equations with long water wave

\[
\begin{align*}
\psi_t - uu_x - v_x + \frac{1}{2} u_{xx} &= 0, \\
v_t - (uv)_x - \frac{1}{2} v_{xx} &= 0,
\end{align*}
\]

the symmetries and conversation laws of Eqs. (11) and (12) had been researched\(^{[6,7]} \) In this section, we mainly consider Eqs. (9) and (10), which are general forms of Eqs. (11) and (12).