CONSTRUCTION OF HIGH-ORDER ACCURACY IMPLICIT RESIDUAL SMOOTHING SCHEMES*

Ni Mingjiu, Xi Guang, Wang Shangjin (School of Energy and Power Engineering, Xi'an Jiaotong University, Xi'an 710049, P.R. China)

Abstract: Referring to the construction way of Lax-Wendroff scheme, new IRS (Implicit Residual Smoothing) schemes have been developed for hyperbolic, parabolic and hyper-parabolic equations. These IRS schemes have 2nd-order or 3rd-order time accuracy, and can extend the stability region of basic explicit time-stepping scheme greatly and thus can permit higher CFL number in the calculation of flow field. The central smoothing and upwind-bias smoothing techniques have been developed too. Based on one-dimensional linear model equation, it has been found that the scheme is unconditionally stable according to the von-Neumann analysis. The limitation of Dawes' method, which has been applied in turbomachinery widely, has been discussed on solving steady flow and viscous flow. It is shown that stable solution of this method is not completely independent with the value of time step. In the end, numerical results by using IRS schemes and Dawes' method as well as TVD (total variation diminishing) scheme and four-stage Runge-Kutta technique are presented to verify the analytical conclusions.

Key words: IRS scheme; four-stage Runge-Kutta technique; TVD scheme

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Introduction

General implicit methods (Fu D.X. [1], Hirsch [2]) can be written as
\[ \{\text{Implicit Part}\} = \{\text{Explicit Part}\}. \] (1)

It has been proposed by MacCormack [3] that modern implicit methods can be written as
\[ \{\text{Numerical Part}\} = \{\text{Physical Part}\}. \] (2)

The physical part reflects the change rule of physical parameter in spatial direction, and the numerical part reflects the change rule in time direction, when steady problem was solved, the solution accuracy will be determined by the physical part and the solving efficiency will be determined by the numerical part. The IRS scheme [4] is one of the modern implicit method. In fact, it is a standard technique to accelerate the convergence of explicit time-stepping schemes to

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Biography: Ni Mingjiu (1969 ~ ), Lecturer, Doctor, E-mail: mjni@xjtu 01.xjtu.edu.cn
steady state using IRS together with multi-grid. Blazek et al. \cite{5} presented two kinds of IRS scheme, i.e. central smoothing technique. The later has advantages over the central smoothing schemes. In this paper, IRS schemes for time-dependent equation have been reconstructed referring to the construction method of Lax-Wendroff scheme\cite{6}. It has been found that Dawes’ method\cite{7} can be regarded as a special case of the central weighted IRS scheme developed in this paper and has limitation in solving stable flow and viscous flow. It has been shown that the stable solution by using Dawes’ method is not completely independent with the value of time step. In order to verify the developed IRS schemes, 2-D reflective shock wave problem has been calculated by using upwind-bias scheme and Jameson’s standard IRS scheme and Dawes’ method connected with four-stage Runge-Kutta technique and the Yee’s symmetrical TVD scheme\cite{8,9}.

1 IRS Schemes for Time-Dependent Equations

1.1 Construction of high-accuracy IRS schemes

Consider one-dimensional model parabolic equation

\[ \frac{\partial u}{\partial t} = \varepsilon \frac{\partial^2 u}{\partial x^2}. \]  

We expand the time-term by using Taylor series

\[ u^{n+1} = u^n + \frac{\partial u}{\partial t} \Delta t + \frac{1}{2} \frac{\partial^2 u}{\partial t^2} (\Delta t)^2 + \frac{1}{6} \frac{\partial^3 u}{\partial t^3} (\Delta t)^3 + \cdots. \]  

Substituting Eq. (3) into Eq. (4), we have

\[ u^{n+1} = u^n + \varepsilon \frac{\partial^2 u}{\partial x^2} \Delta t + \frac{1}{2} \varepsilon \frac{\partial^2 u}{\partial x^2} \left( \frac{\partial u}{\partial t} \right) (\Delta t)^2 + \cdots. \]  

Eq. (5) can be discretized as

\[ \left( 1 - \frac{1}{2} \varepsilon \frac{\Delta t}{(\Delta x)^2} D_2 \right) \Delta u^n = \varepsilon \frac{\Delta t}{(\Delta x)^2} D_2 u^n, \]  

where \( \Delta u^n = u^{n+1} - u^n \), and \( D_2 \) is the discretized solver of second-order space derivative term, \( D_2 = (1, -2, 1) \), that means; \( D_2 u_i = u_{i-1} - 2u_i + u_{i+1} \). Eq. (6) is the central weighted averaged implicit residual smoothing scheme for parabolic equation. It is apparent from Eq. (5) and Eq. (6) that the scheme has second-order time and space accuracy.

For one-dimensional model hyperbolic equation

\[ \frac{\partial u}{\partial t} + a \frac{\partial u}{\partial x} = 0. \]  

By using the similar way as for parabolic Eq. (3), we can construct two different IRS schemes, which can be written respectively as

\[ \left( 1 - \frac{\alpha^2 (\Delta t)^2}{6 (\Delta x)^2} D_2 \right) \Delta u^n = - \frac{\alpha \Delta t}{\Delta x} \delta_x u^n + \frac{1}{2} \frac{\alpha^2 (\Delta t)^2}{(\Delta x)^2} D_2 u^n, \]  

\[ \left( 1 + \frac{\alpha (\Delta t)^2}{2 (\Delta x)^2} \delta^*_x \right) \Delta u^n = - \frac{\alpha \Delta t}{\Delta x} \delta_x u^n, \]  

where \( \delta^*_x \) is upwind space discretized solver, \( \delta_x \) is space discretized solver. Eq. (8) is central-weighted Lax-Wendroff IRS scheme, which has third-order time accuracy. Eq. (9) is upwind-bias IRS scheme, it has second-order time accuracy. For a steady problem, the space discretized solver \( \delta_x u^n \) determines the solution accuracy.

Generally, for one dimensional model hyper-parabolic equation