A REASONABLE METHOD FOR CONSTRUCTING GENERAL ELEMENT DM4 OF THICK AND THIN PLATE WITH EFFECTUAL AND REALIABLE NUMERICAL SOLUTIONS

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Abstract: In this thesis, the internal relations between about shear looking, zero energy mode and patch test are studied, and a reasonable method provided for building general element of thick and thin plate with effectual and realiable numerical solution.

Key words: shear looking; zero energy mode; path test; consistent constraint condition; reasonable Kirchhoff constraint

1 On the Shear Locking Analysis—Condition I

It is difficult to build $C^1$ continuing for $w$ fields in the case of thin plate, and most it of is incompatible element, and it is hardly assured to converge. On the other hand, there are lots of moderate-thick plates widely used in the engineering, so Kirchhoff thin plate theory is not used. Building effectual and realiable thick-thin $C^0$ plate becomes a very attractive research subject, some recent reviews about that can be seen in Refs. [1 ~ 4], but there is a lot of work to do in depth.

\[
\begin{align*}
\sigma & = \sum_{i=1}^{4} N_i(\xi, \eta) \sigma_i, \\
\theta_\xi & = \sum_{i=1}^{4} N_i(\xi, \eta) \theta_\xi, \\
\theta_\eta & = \sum_{i=1}^{4} N_i(\xi, \eta) \theta_\eta,
\end{align*}
\]

Fig. 1 4-node element

In Fig. 1 with regard to the 4-node Mindlin plate element, the bilinear deflection $w$ and rotation $\theta$ fields can be expressed as:

\[
w = \sum_{i=1}^{4} N_i(\xi, \eta) w_i,
\]

\[
\theta_\xi = \sum_{i=1}^{4} N_i(\xi, \eta) \theta_\xi,
\]

\[
\theta_\eta = \sum_{i=1}^{4} N_i(\xi, \eta) \theta_\eta,
\]
where \( N_i(\xi, \eta) \) is criterion shape function:
\[
N_i(\xi, \eta) = \frac{1}{4}(1 + \xi)(1 + \eta) (1 + \eta) (1 + \eta) \quad (i = 1, 2, 3, 4).
\] (2)

Thus the bilinear trial function can be equivalently expressed as:
\[
\begin{align*}
\omega &= c_1 + c_2 \xi + c_3 \eta + c_4 \xi \eta, \\
\partial_x & = a_1 + a_2 \xi + a_3 \eta + a_4 \xi \eta, \\
\partial_y & = b_1 + b_2 \xi + b_3 \eta + b_4 \xi \eta,
\end{align*}
\] (3a) (3b) (3c)

where
\[
\begin{bmatrix}
c_1 & c_2 & c_3 & c_4 \\
b_1 & b_2 & b_3 & b_4
\end{bmatrix}
= \frac{1}{4} \begin{bmatrix}
1 & 1 & 1 & 1 \\
-1 & 1 & -1 & 1 \\
-1 & -1 & 1 & 1 \\
1 & -1 & -1 & 1
\end{bmatrix}
\begin{bmatrix}
\partial_{\xi 1} & \partial_{\eta 1} & \omega_1 \\
\partial_{\xi 2} & \partial_{\eta 2} & \omega_2 \\
\partial_{\xi 3} & \partial_{\eta 3} & \omega_3 \\
\partial_{\xi 4} & \partial_{\eta 4} & \omega_4
\end{bmatrix}
\] (4)

The element strain energy is formulated as follows:
\[
U = U_b + U_s = \frac{1}{2} q^T Kq,
\] (5)
\[
U_b = \frac{1}{2} \int_A x^T S_b x dA = \frac{1}{2} q^T K_b q,
\] (6)
\[
U_s = \frac{1}{2} \int_A \gamma^T S_s \gamma dA = \frac{1}{2} q^T K_s q.
\] (7)

The element stiffness matrix \( K = K_b + K_s \), \( K_s \) being bending stiffness matrix, \( K_s \) is shear stiffness matrix, and \( q \) is node displacement vector.
\[
q = [\omega_1 \quad \partial_{\xi 1} \quad \partial_{\eta 1} \quad \cdots \quad \omega_4 \quad \partial_{\xi 4} \quad \partial_{\eta 4}]^T,
\] (8)
\[
x = \begin{bmatrix}
\partial/\partial x & 0 \\
0 & \partial/\partial y \\
\partial/\partial y & \partial/\partial x
\end{bmatrix}
\begin{bmatrix}
\theta_x \\
\theta_y
\end{bmatrix} = L \theta,
\] (9)
\[
\gamma = \begin{bmatrix}
\partial/\partial x & \partial/\partial y
\end{bmatrix}
\begin{bmatrix}
\omega \\
\theta_y
\end{bmatrix} = dw - \theta,
\] (10)
\[
S_b = \frac{Eh^3}{12(1 - \mu^2)} \begin{bmatrix}
1 & \mu & 0 \\
\mu & 1 & 0 \\
0 & 0 & 1 - \mu/2
\end{bmatrix}, \quad S_s = \frac{kEh}{2(1 + \mu)} \begin{bmatrix}
1 & 0 \\
0 & 1 \\
0 & 0
\end{bmatrix},
\] (11)

where \( k = 5/6 \).

We consider the rectangular element which runs parallel to the Cartesian coordinate axis \( x-y \), thus \( \omega \) and \( \theta \) can be written as:
\[
\omega = [1 \quad x \quad y \quad xy] a,
\] (12)
\[
\theta = \begin{bmatrix}
\theta_x \\
\theta_y
\end{bmatrix} = \begin{bmatrix}
1 & 0 & x & 0 & y & 0 & xy & 0 \\
0 & 1 & 0 & x & 0 & y & 0 & xy
\end{bmatrix} \beta.
\] (13)

In the case of thin plate, Kirchhoff condition can be expressed as:
\[
\gamma = dw - \theta = 0,
\] (14)
so
\[
\begin{bmatrix}
0 & 1 & 0 & y \\
0 & 1 & 0 & x & 0 & y & 0 & xy
\end{bmatrix} \beta = 0.
\] (15)