DAMAGE THEORY FOR POLYMERIC MATERIAL *

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Abstract: In this paper, the effects of external environment such as corrosion medium, radiation and non-consistent materials on deformation of polymers are discussed. Damage constitutive equations for non-linear elastic and non-linear viscoelastic polymers and a variational principle for viscoelastic damage are also given.

Key words: polymer; damage; variational principle

1 Effects of External Environment on Deformation of Polymers

Under the mal-condition (such as corrosion medium, radiation and non-consistent materials), the inner structure and mechanical properties of polymers will change such as the fracture of macromolecular chains, crack propagation. We called these as damage in material.

We introduce damage parameter into continuous medium model.

1.1 The damage evolution in the differential form

\[ \dot{\omega} = \Phi(\sigma_{ij}, \varepsilon_{ij}, \varepsilon_{ij}, \omega, t, T, S), \]  

or

\[ \dot{\omega} = f_1(\sigma_{ij}) W f_2(W) f_3(\omega), \]  

in which \( \sigma_{ij} \) and \( \varepsilon_{ij} \) are the stress tensor and the strain tensor respectively. \( T \) is temperature, \( S \) is environmental parameter. \( W \) stands for strain energy, \( f_1, f_2 \) and \( f_3 \) are the functions of stress, of strain energy and of damage respectively. The simple form of Eq. (1) in one-dimensional condition can be written

\[ \dot{\omega} = C \left[ \frac{\sigma(1 + \varepsilon^e)}{1 - \omega} \right]^{m} \frac{\omega^\beta}{(1 - \omega)^\gamma}, \]  

where \( \varepsilon^e \) is non-elastic deformation, \( C, m, q, r \) and \( \beta \) are material parameters.

1.2 The damage evolution in the integral form

\[ \omega = \omega_0 + \int_0^t \Pi(t - \tau) \psi(\omega, I, J) d\tau, \]  

where \( \omega_0 \) stands for initial damage, \( \omega |_{\tau=0} = \omega_0 \), \( \Pi(t - \tau) \) is damage kernel, describing the relation of damage with time and stress; \( \psi(\omega, I, J) \) is a non-linear function of stress, strain and damage; \( I \) and \( J \) are defined as

\[ I^2 = \frac{3}{2} \left( \sigma_{ij} - \frac{1}{3} \varepsilon_{ij} \varepsilon_{ik} \varepsilon_{kl} \right) \left( \sigma_{ij} - \frac{1}{3} \varepsilon_{ij} \varepsilon_{ik} \varepsilon_{kl} \right), \]  

\[ J^2 = \frac{3}{2} \left( \varepsilon_{ij} - \frac{1}{3} \varepsilon_{ij} \varepsilon_{ik} \varepsilon_{kl} \right) \left( \varepsilon_{ij} - \frac{1}{3} \varepsilon_{ij} \varepsilon_{ik} \varepsilon_{kl} \right). \]  

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1.3 Damage tensor

We define the damage derivation tensor \( \tilde{\omega}_{ij} \) and the mean damage \( \omega \) as follows:

\[
\tilde{\omega}_{ij} = \int_0^t K[(t - \tau), \eta(\tau)] \beta(\tau) \sigma(\tau) d\tau,
\]
(5)

\[
\omega = \int_0^t K[(t - \tau), \eta(\tau)] \beta(\tau) \sigma_0(\tau) d\tau,
\]
(6)

where \( \beta_Y = \sqrt{3/2} S_0 / \sigma \) and \( \sigma_0 \) is the mean stress, \( \eta(t) = 3\sigma / \sigma \).

The general expression of damage tensor is

\[
\omega_{ij} = \int_0^t K_{ijkl}[(t - \tau), \eta(\tau)] \beta^{ijkl}(\tau) \sigma(\tau) d\tau,
\]
(7)

in which \( K_{ijkl} \) is a tetradic, also called damage tensor.

As to the amount of damage, we consider

\[
M(t) = \tilde{\omega}(t) + 3\beta_0(t) (M(t) \in [0, 1]),
\]

in which \( \tilde{\omega}^2 = \tilde{\omega}_{ij} \cdot \tilde{\omega}_{ij} \) stands for the damage intensity, \( \alpha \) and \( \beta \) are material parameters.

2 Damage in Non-linear Elasticity in One-Dimensional Condition

Let stress-strain law for non-linear elastic polymer be

\[
\sigma = E(\omega) \varepsilon - m(\omega) \varepsilon^3.
\]
(8)

We assume the damage evolution can be described as

\[
\frac{d\omega}{dt} = \alpha(S) \left( \frac{\sigma}{1 - \omega} \right)^{k(t)}.
\]
(9)

In Eq. (8), \( E(\omega) \) and \( m(\omega) \) are expressed as

\[
E(\omega) = E_0(1 - \omega)^a, \quad m(\omega) = m_0(1 - \omega)^b.
\]
(10)

From here we can see the effect of \( \omega \) on \( E \). Letting \( \sigma = \text{const} \), we obtain the rate of strain

\[
\frac{d\varepsilon}{dt} = \frac{a\sigma^3}{E_0^2 - 3m_0^2\sigma^2(1 - (b + 1)a\sigma^b)t^{(b+1)/b + 1})}.
\]
(11)

In this way, we can describe the effect of surrounding on materials by means of some parameters \( E_0, m_0, a, b, \alpha \).

Letting the constitutive equation be

\[
\sigma = A\varepsilon^a(1 - \omega)^b,
\]
(12)

we can get the strain-rate equation

\[
\varepsilon = \varepsilon_0[1 - (b + 1)a\sigma^b t]^{-a/(m(b+1))},
\]
(13)

Eqs. (11) and (13) are also available for analyzing long-term strength of explosive materials.

3 Damage in Non-linear Viscoelastic Material

We introduce damage parameter into viscoelastic equation,

\[
\frac{s_y}{2G_0} = e_y \varphi(e_i, \omega) - \int_0^t K(t - \tau) \varphi(e_i, \omega) e_y(\tau) d\tau,
\]
(14)

where \( s_y \) is stress deviation, \( e_y \) is strain deviation, \( R(t - \tau) \) is the function of rheologic behavior (also called kernel function or influence function), \( \varphi(e_i, \omega) \) and \( \varphi(\theta, \omega) \) are generalized