RESISTIVE INSTABILITIES IN PLASMA

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Abstract. Instabilities produced by finite-resistivity effects in a plasma are of great interest in connection with research in fusion devices, solar flares, and geomagnetic substorms. We elucidate here the physical mechanism of this instability, and in particular, identify the tendencies in the system towards the instability and the tendencies opposing it, if any. As an illustration, we consider the example of the so-called gravitational interchange mode wherein a plasma with a statically stable vertical density gradient is situated in a vertical gravitational field and a sheared horizontal magnetic field. The physical picture developed here may be useful in sorting out phenomena that appear when more subtle properties of the resistive modes in a plasma are considered.

1. Introduction

Instabilities produced by finite resistivity effects in a plasma (Furth et al., 1963) have assumed considerable importance in fusion research because certain disruptive phenomena in tokamaks are found (Hosea et al., 1971; Mirnov et al., 1971) to have some significant features in common with the predicted properties of such instabilities. These instabilities are also believed to be associated with rapid energy releases in solar flares and geomagnetic substorms (Sonnerup, 1979).

The presence of a finite resistivity in plasma leads to the local relaxation of the constraint that the plasma must remain attached to the magnetic field. This leads to the possibilities of lowering the potential energy of the magnetic field because the presence of a finite resistivity makes some energetically possible modes topologically accessible. In particular, a finite plasma-resistivity allows the magnetic field lines, that are initially distinct, to link up during the perturbation. These modes have no counterparts in the infinite plasma-conductivity limit, and then disappear altogether, because of the basic topological restriction of the infinite plasma-conductivity model that the field lines initially distinct must remain so in the course of a physical perturbation.

The main features of resistive instabilities (also called tearing modes – because the magnetic field lines are torn and reconnected as these instabilities evolve) are:

(i) they are driven by the energy stored in the magnetic field shear;
(ii) they alter the magnetic field topology and quickly relax the magnetic-field shear – this involves magnetic field – line reconnection and magnetic island formation;
(iii) the relaxation of the magnetic-field shear occurs on time-scales much shorter than the resistive diffusion time.

Most of the work on this problem has tended to utilise realistic configurations, where the effort is concentrated on mathematical techniques. Thus, too often one finds that

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although the basic ideas in stability theory are reasonably transparent, the link between them and the mathematics is often obscure. The purpose of this paper is to elucidate the physical mechanism of this instability, and in particular, identify the tendencies in the system toward the instability and the tendencies opposing it, if any. As an illustration, we consider the example of the so-called gravitational interchange mode (Bateman, 1978), wherein a plasma with a statically stable vertical density gradient is situation in a vertical gravitational field and a sheared horizontal magnetic field. This example affords an insight into the physical mechanisms and shows how they enter the mathematical formalism. The physical picture developed here may be useful in sorting out phenomena that appear when more subtle properties of the resistive modes in a plasma are considered.

We will consider the magnetohydrodynamic description to this problem which results by adopting a continuum fluid model for the plasma. In this model the plasma properties are represented by only the mass density $\rho$, pressure $p$, mass velocity $V$, and the electric current $J$. These quantities can be taken to be averages over all the particles in a suitably chosen small volume, or, more elegantly, they represent the average over many macroscopically identical systems with differing microscopic distributions (the ensemble average). The success of the fluid approximation for a plasma depends on whether the information discarded in the process is irrelevant to the subsequent descriptions. The practical significance of the fluid model is not that it is a very good approximation to any real plasma, but rather that it offers the easiest model by which the macroscopic interaction between plasmas and magnetic fields may be studied. One may then visualise the behavior of real plasmas in terms of deviations from this firmly established and quantitative ideal. It turns out that many plasma phenomena can indeed be described by a fluid model. Basically, the fluid approximation is valid if there is sufficient localisation of particles in physical space. This localisation can be accomplished by two means:

(i) collisions between the particles;
(ii) gyrations of the particles in a strong magnetic field.

In the latter case, the localisation is possible only across the magnetic field lines.

2. Basic Equations

Consider an infinite plane current layer along the $x$-axis (Figure 1) in an inviscid and incompressible plasma. The plasma is stratified in the $y$-direction and is subjected to a gravitational field $g$ in the same direction (which may also simulate the effect of a magnetic field curvature). The plasma stratification is such that it is statically stable (i.e., the density increases in the direction of the gravitational force).

The governing equations in the magnetohydrodynamic description are

\[ \nabla \cdot v = 0 , \]  \hspace{1cm} (1)

\[ \frac{\partial p}{\partial t} + (v \cdot \nabla) p = 0 , \]  \hspace{1cm} (2)