ANALYTICAL SOLUTION OF RADIATED SOUND PRESSURE OF RING-STIFFENED CYLINDRICAL SHELLS IN FLUID MEDIUM

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Abstract

In this paper, analytical formulations of radiated sound pressure of ring-stiffened cylindrical shells in fluid medium are derived by means of Hamilton's principle, Huygens' principle and Green function. These formulations can be used to compute the sound pressure of the shell's surface, nearfield and farfield.

Key words ring-stiffened cylindrical shell, radiated sound pressure, Huygens' principle

I. Introduction

Acoustic radiation from ring-stiffened cylindrical shells which are the main structure of submarine hulls under excitation in fluid medium is very important. The study of the harmonic acoustic field radiated from vibrating structure in ideal fluid medium can be reduced to solve a vibro-acoustic coupled system which is comprised of dynamic equations of structure, Helmholtz equation in fluid, Sommerfeld radiation condition and fluid-structure interaction conditions.

Stepanishen\(^{1, 2}\) studied the radiated sound pressure and radiation impedance of cylindrical surfaces with nonuniform velocity distributions. Laulagnet and Guyader\(^{3, 4}\) investigated the acoustic radiation of cylindrical shells and ring-stiffened cylindrical shell respectively, they only gave the general expression of radiated sound pressure of the cylindrical shell's surface that could not be applied in numerical computations directly. In the present paper, the analytical formulas of radiated sound pressure of the ring-stiffened cylindrical shell's surface, nearfield and farfield are obtained, the correctness of the formulas in this paper is verified by the comparison of numerical results and measured sound pressure.

II. Theory

Consider a finite ring-stiffened cylindrical shell, terminated by infinite cylindrical rigid baffles and immersed in an infinite acoustic medium (see Fig. 1). The shell is excited by harmonic point forces.

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The shell's dynamic equation can be derived by using Hamilton's principle, the variation equation is:

$$\delta \int_0^{t_f} \left[ \left( U_s + \sum_{i=1}^{N_s} U_i \right) - \left( V_s + \sum_{i=1}^{N_s} V_i \right) - \left( E_p + E_q + E_r \right) \right] dt = 0$$  \hspace{1cm} (2.1)

Here $U_s$ is the shell deformation energy, $V_s$ is the shell kinetic energy, $U_i$ is the deformation energy of the $i$th stiffener, $V_i$ is the kinetic energy of the $i$th stiffener, $E_p$, $E_q$ and $E_r$ are the work done by sound pressure of the shell's surface, hydrostatic pressure and external excitations respectively.

Since the external excitations, sound pressure and the shell displacements all include the time factor $e^{-i\omega t}$, $e^{-i\omega t}$ is not written in later derivations for briefness.

It is supposed that the shell is simply supported at its ends, the displacements of the submerged ring-stiffened cylindrical shell can be expanded by using the modes of the unstiffened in vacuo shell as follows:

$$u = \sum_{a=0}^{1} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} a_{nm} \left[ D_{nm} \sin \left( n \theta + \frac{a \pi}{2} \right) \cos \frac{m \pi x}{L} \right]$$

$$v = \sum_{a=0}^{1} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} a_{nm} \left[ E_{nm} \cos \left( n \theta + \frac{a \pi}{2} \right) \sin \frac{m \pi x}{L} \right]$$

$$w = \sum_{a=0}^{1} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} a_{nm} \left[ \sin \left( n \theta + \frac{a \pi}{2} \right) \sin \frac{m \pi x}{L} \right]$$  \hspace{1cm} (2.2)

Here $u$, $v$, $w$ are the axial, circumferential and radial components of the displacement of the shell's mid-surface, $a_{nmj}$ is generalized coordinate, $a=0$ (resp. $a=1$) denotes antisymmetric (resp. symmetric) modes, $n$, $m$ are respectively circumferential and longitudinal orders, $j$ is the type of the mode (bending, twisting, extension-compression), $(D_{nmj}, E_{nmj}, 1)$ are the eigenvector components.

According to Huygens' principle, the radiated sound pressure is:

$$P(X) = \int_S G(X | X_s) V_r(X_s) dS_s$$  \hspace{1cm} (2.3)

Here $S$ is the surface of the shell, $X = (x, r, \theta)$, $X_s = (x_p, R, \theta_0)$, $V_r(X_s) = -i \omega w(X_s)$ denotes the radial velocity of the shell's surface, $G(X | X_s)$ is the well-known harmonic Green's function for the exterior Neumann boundary value problem$^5$. 

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**Fig. 1 Ring-stiffened cylindrical shell model**