BAROTHERMIC EFFECT IN FILTRATION
OF ANOMALOUS OIL AND WATER

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The regularities of formation of the temperature field at the exit from a pool are established from
a numerical study of the temperature field due to the barothermic effect in filtration of anomalous
oil and water. It is shown that the anomalous properties of oil enhance the contribution of
adiabatic cooling at the initial stages of production from an oil well. In simultaneous motion of
anomalous oil and water, a nonmonotonic dependence of the fluid temperature on the saturation
of the pool with water is observed.

As is known, the study of the temperature fields due to the barothermic effect is of practical significance.
At present, the temperature fields due to the Joule–Thomson effect and adiabatic effect in the filtration of a
Newtonian fluid have been primarily studied [1–3].

The barothermic effect in the filtration of anomalous fluids have been inadequately studied. Filippov
and Khusainova [4] reported an approximate analytical solution for the problem of the temperature field of
the barothermic effect in the filtration of viscoplastic oil.

The present paper deals with a numerical study of the temperature field due to the Joule–Thomson
effect and the adiabatic effect in the filtration of anomalous oil and water. The mathematical model for the
nonisothermal filtration of anomalous oil and water ignores the diffusion transfer of mass and heat, the mutual
dissolution of oil and water, heat exchange with the ambient medium, and gravity and capillary forces.

1. The mathematical model is based on the equation of conservation of mass for the phases, the
equation of motion, and the heat inflow equation.

With allowance for the above remarks, the equation of conservation of mass for the phases in the
plane-radial case has the form

\[ \frac{\partial m_i \rho_i S_i}{\partial t} + \frac{1}{r} \frac{\partial r m_i \rho_i S_i V_i}{\partial r} = 0, \quad i = 1, 2. \]  

The values 0, 1, and 2 of the subscript i refer to rock, water, and oil, respectively, \( S_i \) and \( V_i \) are the saturation
and velocity of motion of the ith phase, \( \rho_i \) is the gravity of the ith phase, and \( m \) is the porosity.

The equation of motion for water is written in the form of the Darcy filtration law:

\[ m S_1 V_1 = -\frac{K k_1}{\mu_1} \frac{\partial P}{\partial r}. \]  

For the oil phase, we use the law of filtration of a viscoplastic fluid [5]:

\[ m S_i V_i = -\frac{K k_i}{\mu_i} \left( \frac{\partial P}{\partial r} - G \right) \quad \text{for} \quad \frac{\partial P}{\partial r} > G, \quad m S_i V_i = 0 \quad \text{for} \quad \frac{\partial P}{\partial r} < G. \]  

Here \( K \) is the absolute permeability, \( k_i \) is the permeability of the phase, \( \mu_i \) is the viscosity of the ith phase,
\( P \) is the pressure, and \( G \) is the initial shear pressure gradient.
The heat inflow equation in the approximation of a one-temperature model that takes into account the Joule–Thomson effect, the adiabatic effect, and convective heat transfer and ignores thermal conductivity has the form [1, 6]

\[
\frac{\partial}{\partial t} \left[ (1 - m) \rho_0 C_0 T + \sum_{i=1}^{2} m_i \rho_i C_i S_i T \right] + \frac{1}{r} \frac{\partial}{\partial r} \left[ r \sum_{i=1}^{2} \rho_i C_i S_i V_i T \right] + m \sum_{i=1}^{2} \rho_i C_i S_i V_i \frac{\partial P}{\partial r} - m \sum_{i=1}^{2} \rho_i C_i S_i \eta_i \frac{\partial P}{\partial t} = 0. \tag{4}
\]

Here the following thermodynamic parameters are introduced: \( T \) is the temperature, \( C_i \) is the heat capacity, \( \varepsilon_i \) is the Joule–Thomson coefficient, and \( \eta_i \) is the adiabatic coefficient of the \( i \)th phase.

The first term of Eq. (4) represents the change in the heat content of the system, the second term represents convective heat transfer, and the third and fourth terms are the contributions of the Joule–Thomson and adiabatic effects, respectively.

The initial and boundary conditions are

\[
t = 0, \quad r > 0: \quad S_i = S_i^0, \quad P = P_0, \quad T = T_0,
\]
\[
t > 0, \quad r = R_0: \quad P = P_K(t), \quad P_K^0 \leq P_K(t) \leq P_0,
\]
\[
t > 0, \quad r = R: \quad P = P_0, \quad S_i = S_i^0, \quad T = T_0. \tag{5}
\]

The initial and boundary conditions were specified in the same manner as in [7]. The oil and water densities are functions of pressure and temperature.

System (1)-(4) with initial and boundary conditions (5) was solved numerically using a conservative finite-difference scheme of through calculation. The saturations of the phases and the temperature were calculated by an explicit scheme, and the pressure was calculated by an implicit scheme. Testing was performed using the known analytical solutions of the temperature field due to the Joule–Thomson effect for the filtration of Newtonian oil [1].

2. The calculations were performed for the following model values of the thermohydrodynamic parameters of the phases, which are close to the real rock values [1, 8]:

\[
C_0 = 800 \text{ J/(kg} \cdot \text{K)}, \quad C_1 = 4000 \text{ J/(kg} \cdot \text{K)}, \quad C_2 = 2000 \text{ J/(kg} \cdot \text{K)},
\]
\[
\varepsilon_1 = 0.2 \text{ K/MPa}, \quad \varepsilon_2 = 0.4 \text{ K/MPa}, \quad \eta_1 = 0.015 \text{ K/MPa}, \quad \eta_2 = 0.13 \text{ K/MPa}.
\]

The initial rock pressure \( P_0 \) and the minimum pressure \( P_K^0 \) at the rock boundary \((r = R_0)\) are equal to 20.0 and 14.0 MPa, respectively.

The calculations were performed for the following initial shear pressure gradients [9]: \( G = 0, 0.02, \) and \( 0.05 \text{ MPa/m} \). The water and oil viscosities were assumed to be \( \mu_1 = 0.1 \text{ mPa} \cdot \text{sec} \) and \( \mu_2 = 0.4 \text{ mPa} \cdot \text{sec} \), respectively. The initial water saturation was varied within \( S_0 = 0, 0.25, 0.5, \) and \( 0.7 \).

3. Figure 1 shows results of calculation of the temperature field for filtration of anomalous oil with various initial shear pressure gradients. Here curve 1 refers to \( G = 0 \), curve 2 to \( G = 0.2 \text{ MPa/m} \), and curve 3 to \( G = 0.5 \text{ MPa/m} \). For \( G = 0 \), the time dependence of temperature is similar to the solution of the problem of the temperature field due to the barothermic effect in the filtration of Newtonian oil [1]. Figure 1 compares the experimental dependence of temperature obtained in survey of well No. 6558 (Bashkiria) (curve 5) with the calculated dependence (curve 4) for \( K/\mu_2 = 0.35 \).

The curve of temperature versus time shows the characteristics portions associated with the development of the adiabatic effect and the Joule–Thomson effect. In the early stages of production from a well (drop of pressure at the exit from the pool), adiabatic cooling of the oil is observed. Further, because of the predominance of throttling heating over adiabatic cooling, the temperature increases and a positive stationary temperature is established. In filtration of anomalous oil, the formation of the thermal field at the exit from the pool is similar to the case of filtration of Newtonian oil. However, with increase in the initial pressure gradient, the contribution of adiabatic cooling increases (curves 2 and 3 in Fig. 1), and the