ONE-DIMENSIONAL MOTION OF AN EMULSION WITH SOLIDIFICATION

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A mathematical model is proposed for the process of solidification of an emulsion with a small disperse-phase concentration moving under the action of thermocapillary forces and microgravity. The first-approximation problem that arises when solutions are represented as asymptotic series in a small parameter is examined. Conditions for the partial and complete displacement of the impurity from the solidified part and conditions for the accumulation of the impurity in the solidified mixture are obtained. The problem of producing a composite with a specified disperse-phase distribution is considered. Exact solutions that adequately reflect various features of the qualitative behavior of the general solution under different input data are obtained and examined.

1. FORMULATION OF THE PROBLEM

1.1. Basic Assumptions. (1) The one-dimensional motion of an emulsion under the action of thermocapillary forces and microgravity is described by the mathematical model proposed in [1]. The required quantities are the temperature \( T \) and concentration of the disperse phase \( c \). After they are determined, the rate and pressure are obtained from additional equations.

2. The process of solidification is described within the framework of the classical Stefan problem ignoring the jump in density during solidification.

3. The solidified matrix is immovable, and hence, the volume average rate of motion of the mixture is equal to zero.

Under the above assumptions, the temperature of the mixture and the concentration of the disperse phase of the liquid mixture are defined by the equations

\[
\begin{align*}
\frac{\partial}{\partial t} c + \frac{\partial}{\partial x} \left( c(1 - c) \left( L \frac{\partial T}{\partial x} + Kg \right) \right) &= 0, \\
\rho_d \lambda_d c \left\{ \frac{\partial T}{\partial t} + (1 - c) \left( L \frac{\partial T}{\partial x} + Kg \right) \frac{\partial T}{\partial x} \right\} + \rho_m \lambda_m (1 - c) \left\{ \frac{\partial T}{\partial t} - c \left( L \frac{\partial T}{\partial x} + Kg \right) \frac{\partial T}{\partial x} \right\} &= k_m \frac{\partial}{\partial x} \left( (1 - Mc) \frac{\partial T}{\partial x} \right). 
\end{align*}
\]

Here \( M = 3(k_m - k_d)/(2k_m + k_d), \) \( K = 2R^2(\rho_d - \rho_m)(\mu_d + \mu_m)/(3\mu_m(2\mu_m + 3\mu_d)), \) and \( L = 2Rk_m \sigma \mu_m/(2\mu_m + 3\mu_d)(2k_m + k_d) \), where \( \rho_d, \rho_m, \lambda_d, \lambda_m, \mu_d, \mu_m, k_d, \) and \( k_m \) denote the density, heat capacity, dynamic viscosity, and thermal conductivity in the disperse (subscript \( d \)) and carrier (subscript \( m \)) phases (these coefficients are considered positive and constant), \( R \) is the radius of disperse particles, \( \sigma \) is a positive constant that is the coefficient of the linear dependence of surface tension on temperature, and \( g \) is the constant acceleration of
microgravity, which is considered positive if the acceleration of gravity is directed toward higher temperature; otherwise, it is negative.

In the liquid matrix with solidified inclusions, there is no thermocapillary effect, and, therefore, in Eq. (1.1), one should set \( L = 0 \). Finally, in the solid matrix there are neither the thermocapillary effect nor Archimedes force, and, hence, \( L = K_g = 0 \). The conditions on a line of strong discontinuity have the form

\[
[c]D = [c(K_g + LT_x)(1 - c)], \quad [T] = 0,
\]

where \([.]\) denotes the difference between the function values on different sides of the discontinuity line, the jump in internal energy \([U]\) is considered constant and equal to \( \gamma \), and \( D \) is the velocity of the phase boundary.

We introduce the "reduced concentration" \( C = c/\varepsilon \), where \( \varepsilon \) is the maximum value of the initial concentration \( (\varepsilon \ll 1) \). The new function will be called by the former name, and its values are no longer restricted from above by unity.

Equations (1.1) and (1.2) become

\[
\frac{\partial}{\partial t} C + \frac{\partial}{\partial x} \left( C(1 - \varepsilon C) \left( L \frac{\partial T}{\partial x} + K_g \right) \right) = 0,
\]

\[
\rho_d \lambda_d \varepsilon C \left\{ \frac{\partial T}{\partial t} + (1 - \varepsilon C) \left( L \frac{\partial T}{\partial x} + K_g \right) \frac{\partial T}{\partial x} \right\} + \rho_m \lambda_m (1 - \varepsilon C) \left\{ \frac{\partial T}{\partial t} - \varepsilon C \left( L \frac{\partial T}{\partial x} + K_g \right) \frac{\partial T}{\partial x} \right\} = k_m \frac{\partial}{\partial x} \left( (1 - \varepsilon C) \frac{\partial T}{\partial x} \right),
\]

and the strong discontinuity conditions are

\[
[C]D = [C(K_g + LT_x)(1 - \varepsilon C)], \quad [T] = 0,
\]

\[
[U]D = [(\rho_d \lambda_d - \rho_m \lambda_m)\varepsilon C(1 - \varepsilon C)(K_g + LT_x)T] - [k_m(1 - \varepsilon C)T_x].
\]

1.2. Additional Assumptions. The assumption of low disperse-phase concentration gives ground for linearization, which is the main method of studying the problem in the present work. Since thermal conduction as a first approximation \( (\varepsilon = 0) \) is determined by the parameters of the carrier phase (matrix), we assume that:

— The boundary of solidification of the matrix \( x = s_m(t) \) is a Stefan boundary for the thermal problem, and the conditions of strong discontinuity for the temperature and concentration are satisfied on the boundary;

— The isotherm \( T = T^d \), where \( T^d \) is the solidification temperature of the disperse phase, can be a strong discontinuity line only for the disperse-phase concentration \( [x = s_d(t) \) is treated as the equation of this isotherm];

— The heat flux is directed toward increasing the \( x \) coordinate.

Let the solidification temperature of the matrix \( T_m \) be lower than the solidification temperature of the disperse phase \( T^d \). The concentration of the solid disperse phase in the solidified matrix [the region \( x < s_m(t) \)] is denoted by \( C^s(x, t) \), the concentration of the solid disperse phase in the liquid matrix (the region \( s_m < x < s_d(t) \)) is denoted by \( C^{st}(x, t) \), and, finally, the concentration of the liquid disperse phase in the liquid matrix [the region \( x > s_d(t) \)] is denoted by \( C^l(x, t) \). As a result of expansion in the small parameter \( \varepsilon \), the problem of determining the functions of the first approximation \( C^s(x, t), C^l(x, t), \) and \( C^{st}(x, t) \) takes the form

\[
\frac{\partial}{\partial t} C^l + \frac{\partial}{\partial x} \left( C^l \left( L \frac{\partial T^l}{\partial x} + K_g \right) \right) = 0, \quad x > s_d(t),
\]

\[
\frac{\partial}{\partial t} C^{st} + \frac{\partial}{\partial x} (C^{st} K_g) = 0, \quad s_m(t) < x < s_d(t),
\]

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