LOCAL INSTABILITY OF THE WALLS OF BOREHOLES IN DRILLING IN COMPRESSIBLE HARDENING ELASTOVISCOPLASTIC MEDIA

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The local instability of borehole walls in complex compressible media is studied within the framework of exact three-dimensional equations. Numerical experiments were performed for particular materials. The effect of the dilatancy rate, the viscosity, the gravity parameter, and Poisson's ratio on the critical parameters is estimated.

It is well known that the solution of the problems of rock mechanics related to drilling of oil and gas boreholes is reduced to the formulation and solution of the problems of local instability of the rock zone adjacent to the shaft in the presence of elastoplastic deformations [1-4]. This is due to the fact that even at depths smaller than 1 km, the stresses around vertical workings and boreholes exceed the ultimate strength of the rock; this results in inelastic deformation before the local loss of elastic stability occurs. Evidently, to study the problem of stability of a rock working, one should use more complicated models that describe the behavior of rocks most adequately [4]. In this paper, in contrast to [3], the local instability of the rock in the shaft zone is modeled by relations of a compressible elastoviscoplastic body with translational hardening [5-7].

In this case, the loading function is written in the form

\[ F = \alpha \sigma + \sqrt{(S_i^i - c(\dot{\varepsilon}_i^i)^p - \eta(\dot{\varepsilon}_i^i)^p')(S_i^i - c(\dot{\varepsilon}_i^i)^p' - \eta(\dot{\varepsilon}_i^i)^p')} - \sqrt{2K}, \]

and the relations of the associated flow law have the form

\[ (\dot{\varepsilon}_i^p) = \xi \left( \frac{\alpha}{3} \dot{\varepsilon}_i^j \frac{S_i^j - c(\dot{\varepsilon}_i^j)^p - \eta(\dot{\varepsilon}_i^j)^p'}{\sqrt{2K - \alpha \sigma}} \right). \]

Here \( \alpha \) is the dilatancy rate, \( c \) is the hardening coefficient, \( \eta \) is the viscosity coefficient, \( K \) is the yield point, \( (\dot{\varepsilon}_i^p) \) and \( (\dot{\varepsilon}_i^p)' \) are the deviatoric plastic-strain and plastic strain-rate tensors, \( \sigma = (1/3)S_k^k \), \( S_i^i = \sigma_i^i - \sigma_i^j \) is the deviatoric stress tensor, \( \delta_i^j \) is the Kronecker symbol, \( q^p \) is the fundamental tensor, and \( \xi \) is a positive factor. The subscripts and superscripts \( i, j, \) and \( k \) run from 1 to 3 and the superscript \( p' \) denotes the deviatoric part of the tensor in the plasticity region. Summation over the repeated indices is performed. Relation (2) takes into account the associated compressibility of the material, which is related to the occurrence of plastic shear strains in the body.

A stability analysis of the prebuckling state of a body of volume \( V \) which is characterized by the displacement vector \( \hat{u}_i(x_k, t) \), the stress tensor \( \hat{\sigma}_i^j(x_k, t) \), and the vector of the body \( \hat{X}_i \) and surface \( \hat{P}_i \) forces reduces to the solution of a system of differential equations in variations under the corresponding boundary conditions [8].

The equations of equilibrium for the plastic \( V^p \) and elastic \( V^e \) regions have the form

\[ \nabla_i(\sigma_i^j + \delta_i^j \nabla^\alpha u_j) + X_j - \rho S^i u_j = 0, \quad s = i\omega, \]
where the symbol $\nabla$ denotes covariant differentiation.

The boundary conditions at the outer surface $S_0^p$ (and, hence, $S_0^e$) are

$$(\sigma_j^i + \delta_j^i \nabla^a u_j) n_i = p_j,$$

(4)

Here we have $p_j = \hat{p}_k \nabla^k u_j$ and $X_j = \hat{X}_k \nabla^k u_j$ in the case of a “follower” load and $p_j = X_j = 0$ in the case of a “dead” load. Here and henceforth, the superscripts $p$ and $e$ refer to quantities corresponding to the plastic and elastic regions, respectively, and the circle atop refers to the components of the unperturbed prebuckling state.

The relationship between the amplitude values of the stresses and displacements in the plastic and elastic regions can be written in the form

$$\sigma_j^i = a_{ia} g^{aa} \nabla^a u_k a_j^i + (1 - g^i_j) g^{ii} G_j^i (\nabla^i u_j + \nabla^j u_i)$$

(no summation over $i$ and $j$). In the plastic region, the coefficients $a_{ia}$ and $G_j^i$ have the form

$$a_{ia} = \frac{E}{1 + \nu} \delta_{ia} + \frac{\hat{E}}{D a a} B_{ii} + A_{ii}, \quad G_j^i = \frac{E}{2(1 + \nu)} = G$$

(no summation over $i$ and $\alpha$), where

$$A_{ij} = \frac{E}{1 + \nu} \left( \frac{1}{3} \delta_{ij} - \frac{A B}{(a a)^2 D} \lambda_{ij} - \frac{i_{ij}^i}{a a} \right), \quad B_{ij} = \frac{E}{1 + \nu} \left( \frac{\hat{E}}{D a a} + \frac{C}{a a} \right) \lambda_{ij}, \quad \alpha = \sqrt{2K - \alpha a},$$

$$\lambda_{ij} = \frac{3 f_{ij}(2\nu - 1)}{a a E} - \frac{\nu + 1}{E} \delta_{ij}, \quad A = \delta_{ij} \delta_{ij}, \quad B = 1 + \frac{1 + \nu}{E} C, \quad C = c + s^2,$$

$$D = \frac{1 + \nu}{E} - \frac{3(2\nu - 1) B A}{(a a)^2 E}, \quad \hat{E} = 1 - \frac{3A C(2\nu - 1)}{(a a)^2 E}, \quad \delta_{ij} = \delta_{ij} - \delta_{ij}^i.$$  

In the elastic region, the coefficients $a_{ia}$ and $G_j^i$ are determined by relations (6) for $A_{ij} = B_{ij} = 0$, i.e.,

$$a_{ia} = (\lambda + 2\mu) g_{ia}, \quad G_j^i = \mu.$$  

(7)

In Eqs. (6) and (7), $\lambda$ and $\mu$ are the Lamé parameters, $E$ is Young’s modulus, and $\nu$ is Poisson’s ratio.

We note that the representation (5) is possible only if the prebuckling state is uniform or depends on one variable.

The continuity conditions at the elastoplastic boundary $\Gamma$ have the form

$$[(\sigma_j^i + \delta_j^i \nabla^a u_j)n_j] = 0, \quad [u_i] = 0.$$  

(8)

Equations (3)-(8) form a closed system of equations for analysis of stability problems where a boundary exists between the regions of elastic and plastic behavior of the material during loading.

Let a round borehole drilled vertically in rock be filled with a liquid of density $\gamma$ and its walls be subjected to the pressure $q = \gamma h$, where $h$ is the depth.

The pressure $q$ is called the backpressure of a drilling fluid that prevents the change in shape and dimensions of the cross section of the boreholes. We model [9] the rock mass with a borehole by a weightless infinite plane with a round hole of radius $a$ whose contour is loaded by the uniformly distributed pressure $q$.

At infinity, the stresses in the plate tend to $\gamma_n h$, where $\gamma_n$ is the density of the rock. The stress distribution in the unperturbed rock mass is assumed to be hydrostatic: $p = \gamma_n h$.

In determining the stress and strain components in the prebuckling state in the axisymmetric case, all the functions are written in the form of series in powers of the parameter $\alpha$, i.e., the dilatancy rate:

$$\{\sigma_{ij}, e_{ij}, e_{ij}^p, e_{ij}^n, \xi, \ldots\} \sim \sum_{n=0}^{\infty} \alpha^n \{\sigma_{ij}^{(n)}, e_{ij}^{(n)}, e_{ij}^{(n)}, \xi^{(n)}, \ldots\}.$$