The effect of the shape of the strain-softening diagram on the bearing capacity of concrete beams

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The effect of the shape of the strain-softening diagram on the bearing capacity of three-point loaded beams, as calculated by the "fictitious crack model", is analysed. It is shown that if, at peak load, the openings at the nodes along the fracture zone are smaller than the displacement corresponding to the break-point of the bilinear softening approximation, the maximum load only depends on the slope of its initial linear portion. A "limit break-point" (L) is defined such that bilinear strain-softening diagrams with the same initial slope, and whose break-points are located beyond Point L, will lead to the same maximum load of bent beams. The location of that "limit break-point" as a function of initial slope, beam size and notch depth is also studied; the effect of these variables is explained in terms of notch sensitivity.

1. INTRODUCTION

The "fictitious crack model" (FCM), developed by Hillelberg and his collaborators [1-5] at the Lund Institute of Technology (Sweden), provides a useful tool for predicting the behaviour of either notched or unnotched specimens subjected to tensile stress fields. This already well-known approach is based on the assumption that a fracture zone starts propagating at a point where the first principal stress reaches the tensile strength of the material. Although the material inside the fracture zone is partially damaged, it is still able to transfer stresses, its stress-transferring capability depending on the opening w of the zone or "fictitious crack" in the stressed direction.

The strain-softening behaviour is described by a \( \sigma-w \) relationship. The deformational properties of the material outside the fracture zone are described by a \( \sigma-\varepsilon \) curve, which in most cases can be considered linear elastic.

In order to facilitate the calculations, the \( \sigma-w \) relation obtained in a direction tension test is approximated as piecewise linear. The area under the \( \sigma-w \) curve represents the amount of energy required to create one unit of crack area and, according to the FCM, this is regarded as a material property, the fracture energy \( G_f \).

It has been pointed out by Modéer [2], Gustafsson [5] and Roelfstra and Wittmann [6] that the shape of the strain-softening diagram strongly influences the response of a member, modelled by the FCM, even when the area enclosed by the diagram is kept constant.

Modéer [2] and Gustafsson [5] have mentioned that, at the moment of ultimate load, the fracture zone is only partially developed and, hence, the slope of the first part of the \( \sigma-w \) diagram is essential to the ultimate load whilst the last part of the diagram mainly influences the ductility of the specimen. Gustafsson also comments that this statement is not of general validity but it depends on the size of the specimen considered.

Roelfstra and Wittmann [6], when dealing with the bilinear \( \sigma-w \) approximation, studied the effect of the position of the break-point on the resulting load-displacement diagrams of beams. For that purpose they chose 36 break-points, as shown in Fig. 1b; in all cases the value \( w_c \) was chosen so as to keep \( G_f \) constant. The calculated load-displacement curves are also shown in Fig. 1a. As can be seen, the maximum load as well as the post-peak behaviour of the load-displacement curves are affected by the choice of the break-point. They concluded that the knowledge of \( G_f \) was not sufficient to define the load-deformation behaviour of the beams, a conclusion that is also supported by other authors [7,8].

In order to isolate the influence of the different variables involved, Alvaredo [9] made a similar study but locating 30 break-points along lines of constant slope, whose pattern is shown in Fig. 2b. Again the \( w_c \) values were chosen so as to keep \( G_f \) constant. The load-displacement curves obtained for the 30 break-points, for a 100 mm deep, notched beam (notch/depth ratio = 0.5), are presented in Fig. 2a. It can be noticed that, for bilinear approximations having the same initial slope, the maximum load is the same, whilst the location of the break-point on a certain initial slope seems to affect only the post-peak response.

Taking into account that, usually, the most important target a model must achieve is the adequate prediction of the maximum load, it was decided to investigate more thoroughly the above mentioned behaviour.
2. OBJECT OF THE WORK

The aim of this work is to analyse the influence that the shape of the $\sigma$-$w$ diagram exerts on the bearing capacity of three-point loaded beams with and without notches. The work is based on the hypothesis that if, when the maximum load is reached, the opening at the critical node $k$ (Fig. 3a) is smaller than or equal to the break-point displacement $\beta w_c$, the maximum load would only depend on the initial slope of the piecewise linear strain-softening approximation. In such a case, the choice of different break-points lying on the same initial slope would only affect the post-peak response (Fig. 3b).

Hence, if the openings of the nodes within the fracture zone do not exceed the break-point displacement $\beta w_c$ up to the maximum load, the stress-transferring capability of the fracture zone depends only on the first linear portion of the $\sigma$-$w$ approximation, and it would be insensitive to the assumptions considered for the rest of the strain-softening curve when predicting the specimen behaviour in the load range $\{0-F_{\text{max}}\}$.