Justification and refinements of Model B3 for concrete creep and shrinkage

1. Statistics and sensitivity

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Model B3 for creep and shrinkage prediction in the design of concrete structures, presented as a RILEM Recommendation in Mater. Struct. 28 (1995) 357-365, is calibrated by a computerized data bank comprising practically all the relevant test data obtained in various laboratories throughout the world. The coefficients of variation of deviations of the model from the data are distinctly smaller than for the latest CEB model, and much smaller than for the previous ACI model (which was developed in the mid-1960's). The effect of concrete composition and design strength on the model parameters is identified as the main source of error. The model is simpler than the previous models (BP and BP-KX) developed at Northwestern University, yet it has comparable accuracy and is more rational.

1. INTRODUCTION

Realistic prediction of concrete creep and shrinkage is of crucial importance for the durability and long term serviceability of concrete structures, and in some cases also for the long term stability and load capacity. Mispredictions of this phenomenon, which contribute to excessive deflections and cracking, have been one of the important reasons for problems with longevity of the civil engineering infrastructure in all countries. Errors in the prediction of concrete creep and shrinkage have generally been larger than those caused by simplifications in the methods of structural analysis. It is now clear that, for creep sensitive structures, it makes little sense to use finite element analysis or other sophisticated computational approaches if a realistic model for creep and shrinkage is not introduced in the input. If a simplistic and grossly inaccurate prediction model for creep and shrinkage is used for a creep sensitive structure, one can hardly justify anything more than simple hand calculations of stresses and deformations in structures. In such a case it makes no sense for the analyst to spend weeks on the structural analysis while spending half an hour to determine creep and shrinkage properties to use as the input. The design will be better if more time is devoted to the latter than the former.

Realistic prediction of creep and shrinkage of concrete is a formidably difficult problem because the phenomenon is a result of several interacting physical mechanisms and is influenced by many variable factors. In view of this fact it is not surprising that improvements have been coming only slowly, gradually. No major breakthrough has occurred in the history of the research of this phenomenon; however, the accumulated advancement of knowledge since the early systematic researches in the 1930's, and especially during the last two decades, has been enormous. It is now possible to formulate a much better prediction model than two decades ago.

Four major advances have made a significant prediction improvement possible:

1. Improved theoretical understanding, for example mathematical modelling of the solidification process of cement, diffusion processes, thermally activated processes, cracking damage, residual stresses, and non-uniformity of stress and pore–humidity profiles.

2. Gradual accumulation of test data and formulation of an extensive computerized data bank, which started with the Northwestern University data bank in the 1970's having over 10,000 data points and, in collaboration with ACI and CEB, resulted in the RILEM Data Bank, compiled by Subcommittee 5 of RILEM Committee TC-107 (chaired by H. Müller). (This data bank now comprises about 600 measured time curves from about 100 test series in various laboratories around the world, with about 15,000 data points.)
3. Progress in statistical evaluation of test data and optimization of the creep and shrinkage prediction model, and optimization which minimizes the sum of the squares of errors. This task has been facilitated by the computerized form of the aforementioned RILEM Data Bank.

4. Numerical studies of the response of test specimens and structures (especially by finite elements) and their comparisons with observed behaviour, which has shed light on various assumptions on the material model used in the input.

Much of the complexity and error of the prediction model is caused by the fact that the design offices still analyse most structures according to beam theory, which requires the average material characteristics for the cross-section of the beam as a whole. Because the material creep and shrinkage properties inevitably become non-uniform throughout the cross-section (due to diffusion phenomena, residual stresses, cracking, damage localization and fracture), the model for the average material properties in the cross-section is not a material constitutive model. It depends on many more influencing factors than the constitutive model, e.g., on the shape of the cross-section, environmental history, ratios of the bending moment, normal force and shear force, etc.

For this reason, a really good model for the prediction of the average shrinkage properties and average creep at drying in the cross-section under general loading and environmental conditions will never be possible. One must accept significant errors, increased complexity and a greater number of empirical parameters if one insists on characterizing the behaviour of the cross-section as a whole by its average properties. In the future, the design approach should move away from characterizing the cross-section creep and shrinkage properties of the cross-section as a whole, and towards an approach in which the cross-sections are subdivided into individual small elements. However, for the time being, an integral prediction model for the cross-section as a whole is needed.

In the special case of constant temperature and constant moisture content, the average characterization of creep in the cross-section is identical to a constitutive model for a material point. In this case, the model becomes far simpler and is more accurate.

The present model, which is for brevity labelled model B3 and is based on a recent report [1], is the third major refinement in a series of models developed at Northwestern University, beginning with the BP model [2] and BP-KX model [3, 4]. Model B3 is simpler, better supported theoretically and as accurate as these previous models. Research progress will not stop, and no doubt further improved versions will become possible in the future. Compared with the latest BP-KX model, the improvement in model B3 consists of simplification of the formulae achieved by sensitivity analysis, incorporation of a theoretically derived rather than empirical expression for the drying creep, and calibration of the model by an enlarged data set including the data published in the last few years.

Model B3 conforms to the guidelines that have been formulated recently by RILEM Committee TC-107 [5a], as a refinement and extension of the conclusions of preceding RILEM Committee TC-69 [5b]. These guidelines summarize the basic properties of creep and shrinkage that have been well established by theoretical and experimental research and represent the consensus of the Committee. The existing prediction models of major engineering societies violate many of these guidelines. The present model must nevertheless be regarded as only an example of a model satisfying these guidelines, because formulation of a model based on the guidelines is not unique and conceivably partly different models satisfying these guidelines could be formulated, too.

The purpose of this paper is to provide a justification for, as well as some refinements of, Model B3. It may be noted that the same model as described here has been proposed to ACI Committee 209, and has been approved by the Committee's vote. All the notations from the preceding RILEM Recommendation [6] are retained.

2. UNBIASED STATISTICAL EVALUATION BASED ON THE COMPUTERIZED DATA BANK

The development of a data bank comprising practically all the relevant test results on creep and shrinkage of concrete obtained in various countries and laboratories up to the present time facilitates the evaluation and calibration of creep and shrinkage prediction models. No longer does tediousness limit such evaluation to a few subjectively selected test data. It is important to use the complete set of available test data, because subjective selections of some data for verification of a creep model have been shown capable of greatly distorting the conclusions.

The statistical evaluation and optimization of model B3 have been carried out in a similar manner as for the preceding models BP and BP-KX (see [2] part 6, [3] and [7]). Optimum values of the model parameters which minimize the sum of squared deviations from the data in the data bank have been determined. The deviations of the model from the test data (errors) have been characterized by their coefficient of variation \( \bar{\sigma} \) which is defined for the data set number \( j \) as

\[
\bar{\sigma}_j = \frac{s_j}{\bar{J}_j} = \frac{1}{\bar{J}_j} \left[ \frac{1}{n-1} \sum_{i=1}^{n} (w_{ij} \Delta y_i)^2 \right]^{1/2} \tag{1}
\]

in which

\[
\bar{J}_j = \frac{1}{n} \sum_{i=1}^{n} w_{ij} \bar{J}_i, \quad w_{ij} = \frac{n}{n \sigma n_1} \tag{2}
\]