Shakedown with softening in reinforced concrete beams

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Critical softening parameters for first-formed hinges at any location in a two-span beam are derived, and the relationship between positive load increments and softening parameters less than critical for midspan or interior support hinges are also determined. The effect of non-critical softening on the well known static collapse and shakedown loads for an elastic-plastic beam is found. It is shown that the combined effect of softening and residual moments (such as caused by differential settlements) on the static collapse and shakedown loads may be dramatic.

INTRODUCTION

The failure load for redundant reinforced concrete structures has been defined as that corresponding to a selected upper limit on either steel or concrete strains, thus limiting the permissible rotation of any hinge region[1]. The maximum or collapse load is a less arbitrary definition of the failure load[2], and possibly will not occur until one or more hinges has softened, that is, entered the descending portion of the moment-curvature ($M-\varphi$) curve (fig. 1).

From static load tests ([3], [4]) it is clear that for some structures first-formed hinges may shed a substantial proportion of their maximum moment as their rotation increases and the structure as a whole maintains load capacity. Difficulties with analysis by standard bending theory to include softening may be overcome by employing finite discontinuity lengths or contamination lengths over which softening is assumed to occur ([2], [5], [6], [8]).

Critical softening for a hinge or hinges is defined as that value of negative stiffness on the falling branch of the moment-curvature curve at which the structure as a whole cannot sustain increased load(s), however redundant the structure may still be. Critical softening parameters have been derived for fixed-fixed beams ([5], [7]) and for more general continuous beams and frames [8], using a trilinear approximation to the typical $M-\varphi$ curve (fig. 1).

The author believes that the computation of collapse loads for reinforced concrete frame structures under severe repeated (and possibly reversible) loads or displacements will involve detailed consideration of the softening portion of $M-\varphi$ curves. For this reason a preliminary investigation is made in this paper of the probable influence of softening on static collapse and shakedown loads for continuous beams.

CRITICAL SOFTENING AT ANY POINT IN A TWO-SPAN BEAM

Figure 2 is the basis for the determination of the critical softening parameter for a first-formed hinge at any location within a two span beam. The bending moment increment shown is consistent with no change in external loads, that is, continued deformation occurs without load change. Softening occurs over a hinge length $2l_p$ (chosen to correspond to $l_p$ each side of a maximum moment point). Within the softening region the curvature change is of opposite sign to that of the adjacent region with the same sign of bending moment.

Fig. 1. – Moment-curvature relationship, reinforced concrete section.
Increment. Loading or unloading is everywhere elastic, except within the softening region.

The compatibility equations (no displacement at B or C) are:

\[ \int_0^L \phi (L-x) \, dx + \theta L = 0, \]
\[ \int_0^{2L} \phi (2L-x) \, dx + 2\theta L = 0, \]

where \( \theta \) is the rotation at end A.

Substituting for \( \phi \) in the compatibility equations, and performing the integrations, two equations for \( a_{cr} \) are obtained:

\[ a_{cr} = \frac{4 \left[ 3 \frac{m}{2} (s^3 - s^2) + m (3 - 6 s) - 4 \right]}{m^3 - 4 \left[ 3 \frac{m}{2} (s^3 - s^2) + m (3 - 6 s) - 4 \right] + (6 \theta EI m^3/b L)}, \]
\[ a_{cr} = \frac{2 \left[ 3 \frac{m}{2} (2s^3 - s^2) + 6m (1-s) - 4 \right]}{3m^3 - 2 \left[ 3 \frac{m}{2} (2s^3 - s^2) + 6m (1-s) - 4 \right] + (6 \theta EI m^3/b L)}, \]

where \( m = L/l_p \) and \( s = S/L \) from which:

\[ \frac{6 \theta EI}{b L} = \frac{3m^2 (4s - 5s^2) + 6m (2 - 5s) - 20}{3m^3 s^2 + 6ms + 4}. \]

For large \( m \) (small hinge length), the value of \( \theta \) tends towards:

\[ \theta = b \frac{L}{6EI} \left( \frac{4-5s}{s} \right), \]

which is the rotation at A for equal and opposite couples \( bS/L \) applied at either side of a frictionless point hinge in the two-span beam at location \( S \) (bending moment diagram as in figure 2 b).

Graphs of \( a_{cr} \) for various values of \( m \) and \( s \) are shown in figure 3. The curves stop where the leading edge of the softening region touches the internal support.

A value of \( m \) for any particular case may be calculated by using equivalent hinge lengths \( l_p \) such as determined by Corley [9], or by Sawyer [10].

Softening at a hinge symmetrically placed over the interior support (\( l_p \) each side) is the case of a propped cantilever. The equation, derived elsewhere [8] is:

\[ a_{cr} = \frac{3m^2 - 3m + 1}{m^3 - [3m^2 - 3m + 1]} \]

e. g. for \( m = 20 \), \( a_{cr} = 0.166. \)