THE EFFECT OF ARTERIAL WALL THICKNESS AND CONDUCTIVITY ON ELECTROMAGNETIC FLOWMETER READINGS*

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Abstract—The performance of electromagnetic flowmeters is reviewed in the light of data concerning the tensor conductivity of arterial walls. The muscular structure and its effect on external contact flowmeters is discussed. The situations where the wall effects must be considered are outlined and experimentally demonstrated. Maxwell's equations for the external contact electromagnetic flowmeter are solved considering the difference in conductivity of the wall radially and angularly. The importance of the blood conductivity to the wall conductivity ratio is also demonstrated. It is shown that for situations where the ratio of inside diameter to outside diameter of an artery changes during a flowmeter test, the sensitivity of the flowmeter changes. This is shown to be an important consideration in research studies where vasodilation and vaso-constrictors are used.

The applicability of in vitro flowmeter calibrations to in vivo measurements is also considered.

LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>a</td>
<td>inside radius of the artery</td>
</tr>
<tr>
<td>b</td>
<td>outside radius of the artery</td>
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<tr>
<td>B</td>
<td>magnetic flux density</td>
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<tr>
<td>E</td>
<td>electrical field density</td>
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<tr>
<td>j</td>
<td>current density</td>
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<tr>
<td>K</td>
<td>electrical conductivity of the wall</td>
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<td>(k_1, k_2)</td>
<td>first and second electrical conductivity</td>
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<td>r</td>
<td>radial coordinate</td>
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<tr>
<td>S</td>
<td>(\frac{U_w}{2bBV_m})</td>
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<tr>
<td>U</td>
<td>electrical potential</td>
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<tr>
<td>(\Delta U)</td>
<td>potential difference between electrodes</td>
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<tr>
<td>V</td>
<td>fluid velocity</td>
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<tr>
<td>Z</td>
<td>axial coordinate</td>
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<tr>
<td>(\mu)</td>
<td>permeability</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>electrical conductivity of the fluid</td>
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<tr>
<td>(\theta)</td>
<td>angle relative to the B field</td>
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<tr>
<td>(\tau)</td>
<td>contact resistance between wall and fluid</td>
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<tr>
<td>(\equiv)</td>
<td>second order tensor</td>
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<tr>
<td>(\vec{\nabla})</td>
<td>vector</td>
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Subscripts

<table>
<thead>
<tr>
<th>Subscript</th>
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<tbody>
<tr>
<td>w</td>
<td>wall</td>
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<tr>
<td>f</td>
<td>fluid</td>
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<tr>
<td>m</td>
<td>mean</td>
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1. INTRODUCTION

The electromagnetic flowmeter designed to measure blood flow in arteries has been utilized in many studies both clinically and in experimental research. In the use of this apparatus the calibration is usually performed on an excised artery using either blood or saline and often in a saline bath. This procedure will not be satisfactory in situations where the arterial wall properties and thickness change. These situations may in particular occur when drugs are introduced producing vasodilation and vasoconstriction in an active vascular bed. Kolin (1960) points out that the thickness of the wall of the artery does not effect the flowmeter reading as the conductivity of the wall is nearly the same as that of the blood. This may

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be erroneous if the arterial wall has the properties noted by Burger and Van Dongen (1960) who have shown that the electrical conductivity along and across muscle tissue may be considerably different. They show that the conductivity along the fibers may be roughly ten times that across the muscle fibers. If this is true, (measurements are now in progress to determine more accurately this situation), a correction must be made in flowmeter readings. Similarly, if the calibration is made in a situation where saline in a bath is allowed to permeate the muscle wall the conductivity will be different than in the in vivo situation and an error in the calibration may be observed.

Difficulties in accurate flow measurement and adequate calibration will be encountered primarily in studies where the thickness of the arterial wall is changed considerably. These situations occur primarily in the smaller arteries which are more muscularly active. These effects become very important in flow measurements on a strongly reacting vascular system such as the gastric bed. In these situations several effects can occur which may alter flowmeter readings; (1) a pressure rise in the circulation can cause a thinning of the wall of the artery, (2) a vaso constriction of the artery itself may thicken the wall and, (3) chemical addition can alter the conductivity of the walls relative to the blood.

The effects of wall property changes on the sensitivity of electromagnetic flowmeters have been considered experimentally by both Kolin (1960) and Spencer and Denison (1960). In these experiments, however, chemical and thickness effects were not effectively separated. The data in the experiments noted does not represent the range of conditions which could be encountered in research on small vessels nor those in which the wall thickness is altered appreciably. Spencer and Denison note that placing a vein within an artery and thus doubling the wall thickness does not change the sensitivity. They state that balancing effects are present which cause this to be true. The data available in their paper is not sufficient, however, for analyzing what the balancing effects which occur might be. It might also be noted that most researchers recommend a calibration in vivo at the completion of an experiment. This is not an acceptable procedure for several reasons. First, the wall properties and thicknesses may change, or be altered, during an experiment and, second, the distention and muscular state of the wall will, in general, not be the same if pressures are not also simulated.

In the following analysis the tensor conductivity properties of arterial walls is considered and the sensitivity of flowmeters to wall thickness and property variations examined.

2. ANALYSIS

The usual electromagnetic flowmeter used in medical research is of the transverse field type (Fig. 1).

![Fig. 1. Electromagnetic flowmeter schematic.](image)

The magnetic field is perpendicular to the flow axis of the tube and an e.m.f. $\mathbf{V} \times \mathbf{B}$ is induced by the flow. Electrodes on the outside of the tube perpendicular to the $\mathbf{B}$ field are connected to a voltmeter circuit and the change in voltage with flow is recorded.

Ohm's law may be written for this situation as

$$\mathbf{j} = \mathbf{\sigma} (\mathbf{E} + \mathbf{V} \times \mathbf{B}),$$

where $\mathbf{\sigma}$ is the tensor conductivity of the medium and $\mathbf{j}$ is the current density. $\mathbf{E} + \mathbf{V} \times \mathbf{B}$ is the electric field relative to the moving fluid, and $\mathbf{E}$ is the field due to charges in space.