LETTER TO THE EDITOR

Gompertz Curves, Allometry and Embryogenesis

The claim by Deakin, M. A. B. 1970, ("Gompertz Curves, Allometry and Embryogenesis." Bull. Math. Biophys, 32, 445-452) that "T" represents the time of "embryogenesis" of an organ is not tenable. The basic Gompertz equations are:

\[ y = a \exp \left( -b e^{-rt} \right) \]  \hspace{1cm} (1)

and

\[ Y = A \exp \left( -B e^{-Rt} \right). \]  \hspace{1cm} (2)

These are Equations (12) and (13) of Deakin.

Deakin introduces the parameter, \( T \), by replacing (1) with the following equation:

\[ y = a \exp \left( -b e^{-(r-T)t} \right). \]  \hspace{1cm} (3)

This is Deakin's Equation 14.

The concept of embryogenesis was not mathematically defined, and thus the choice of \( T \) is unrestricted. Since (1) and (2) are equivalent functions, we can write (3) as

\[ y = a \exp \left( -b' e^{-rt} \right), \]  \hspace{1cm} (4)

where

\[ b' = b e^{rT}. \]  \hspace{1cm} (5)

Substituting the logarithmic transforms of (2) and (4) in the allometric law\( \dagger \),

\[ \ln y = \ln k + \alpha \ln Y, \]  \hspace{1cm} (6)

and equating coefficients yield the relationships (16), (17) and (18), Deakin's

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\dagger G. A. Sacher (1970) shows that equation (6) is the fundamental expression of the allometric law.
Equation numbers. For reference, we write the two conditions pertinent to this discussion:

\[ r = R \quad (7) \]

and

\[ \alpha = \frac{b'}{B} \quad (8) \]

Equation (7) is Deakin's (18).

By (5) and (8),

\[ \alpha = \frac{b}{B} e^{rT} \quad (9) \]

Since \( b, B \) and \( r \) are fixed constants and \( T \) is undetermined, \( \alpha \) is a function of \( T \). We write (9) in an equivalent form as,

\[ \ln \alpha = \ln \left( \frac{b}{B} \right) + rT \quad (10) \]

It is seen that \( \ln \left( \frac{b}{B} \right) \) is a fixed constant and is invariant under translations in time. Thus, we set \( T = 0 \) to get

\[ b = \alpha B \quad (11) \]

Let

\[ \alpha = e^{r(\Delta t)} \quad (12) \]

Then

\[ b = B e^{r(\Delta t)} \quad (13) \]

and with \( T = 0 \), (4) becomes,

\[ y = a \exp \left\{ -B e^{-r(t-\Delta t)} \right\} \quad (14) \]

It follows that the condition necessary to satisfy (11) is the equality of initial specific rates of growth; i.e.

\[ \left. \frac{y'(t)}{y(t)} \right|_{t=\Delta t} = \left. \frac{Y'(t)}{Y(t)} \right|_{t=0} \quad (15) \]

By (11) and (12), the lag time is

\[ \Delta t = \frac{1}{r} \ln \left( \frac{b}{B} \right) = \frac{1}{r} \ln \alpha \quad (16) \]

Thus, \( \Delta t \) (Laird et al., 1968) is not a “fictitious time interval,” but measures the constant difference in time between points of equal specific growth rate for (1) and (2). Then, the lag time between two separate organs distinguished by subscripts 1 and 2 is

\[ \Delta t_1 - \Delta t_2 = \frac{1}{r} \ln \left( \frac{\alpha_1}{\alpha_2} \right) \quad (17) \]

the result sought by Deakin's Equation (33).