THE MOTION OF THE LEFT VENTRICLE. I

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The complex arrangement of the muscle fibers in the ventricular wall and the non-
symmetric contraction and expansion of the ventricle preclude the writing of a differential
equation of motion for the ventricle as a whole. We can, however, describe the motion of
the ventricle by describing the motion of the dimensional parameters length and diameter;
the radius, circumference, cross-sectional area, and volume following naturally from these.
The ventricle is assumed to be an ellipsoid of revolution and the dimensional parameters to
be periodic functions of time. Each of the parameters is expressed as a Fourier series.

1. Introduction. The human heart is divided into four chambers, right and
left atria and right and left ventricles, which expand and contract synchronously
to pump the blood throughout the body. The pumping action of the heart
depends upon cyclic changes in the dimensions of the heart chambers. The
great majority of experimental work has been concerned with the ventricles; the
left ventricle having received the most attention due to its rather simple
geometry. The shape of the left ventricle (hereafter called the ventricle) lies
somewhere between the extremes of a sphere and cylinder.

From a theoretical standpoint the ventricle is a vibrating shell in which the
elastic properties are not independent of direction, i.e. the shell is non-isotropic.
A block of tissue cut from the midportion of the wall contains fibers oriented in
three general directions; inner and outer layers are 90 degrees apart—spiralling
in opposite directions and the middle layer encircles the ventricle perpendicular

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to the longitudinal axis (Rushmer, 1961, p. 35). This property along with the notion that the contraction and expansion are not totally symmetric (Hawthorne, 1961) combine to prevent the writing of a differential equation of motion for an element of the shell wall as is usually done in elasticity. The different orientations of the various muscle fibers in the wall give rise to the non-isotropy of the ventricle. However, with the model proposed here we can describe the motion of the ventricle through describing the motion of the parameters length and diameter; the radius, circumference, cross-sectional area, and volume following from these. Certain of these parameters have been measured as functions of time by cardiovascular workers and so it is natural to follow their work with the theoretical tools to describe their data and use it to greater advantage.

Another of the problems that workers in cardiovascular dynamics have not yet resolved is that of describing the volume of the ventricle, in dog and man, as a function of time; Gorlin et al. (1964) have confirmed the need for such a study in their work. Roston (1959) has considered the behavior of the cardiovascular system as a whole by the use of the Laplace transform, which allows mathematical expression of the periodicity of the cardiac output, but from this nothing can be inferred about any of the separate chambers. Robinson (1963) has described a cylindrical model of the ventricle and an analog computer simulation of its behavior. We will propose a model for the ventricle, make some simple assumptions as to its behavior, and then describe the radius, circumference, area, and volume of the ventricle as functions of time given the behavior of the diameter and length as functions of time. Figure 1 shows the experimental records of diameter and length as recorded continuously.

Figure 1. Experimental records of left ventricular diameter (top) and length (bottom) recorded continuously from a dog.