ORGANISMIC SETS AND BIOLOGICAL EPIMORPHISM

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It is shown that the principle of biological epimorphism (Rashevsky, Mathematical Principles in Biology and Their Applications, Springfield, Ill.: Charles Thomas, 1960) is contained in the theory of organismic sets (Bull. Math. Biophysics, 29, 139–152, 1967) if an additional postulate not directly connected to mappings is made.

In another paper (Rashevsky, 1967, hereafter referred to as I) we have outlined the theory of organismic sets which forms a conceptual superstructure over both biology and sociology and brings into focus the relational isomorphisms between biological and social organisms. Still earlier (Rashevsky, 1954; reprinted in Rashevsky, 1960, hereafter referred to as II) we discussed what we then called “the principle of biotopological mapping,” which we later denoted (Rashevsky, 1959) by a more general term of “biological mapping,” and still later (Rashevsky, 1960b), by a “biological epimorphism.” The understanding of the following assumes familiarity with the above mentioned earlier work.

We shall demonstrate that the principle of biological epimorphism is logically contained in the theory of organismic sets if we add to it a postulate. We shall use the same notations as in (I).

An organismic set $S_0$ which consists of $N$ completely unspecialized identical elements, so that $S_i^{(a)} = S_k^{(a)} = S^{(a)}$ for any $i \neq k$, may be considered as a set of $N$ improper organismic sets because each element $e_i$ satisfies in this case all conditions of Definition 1 of (I). Thus each element $e_i$ of such an $S_0$ is itself an organismic set consisting of only one element.

If in line with Definitions 3 and 4 and Postulates 1 and 2 of (I) the elements
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e_i of S_0 undergo specialization, they cease to be identical. The structure of the
set S_0 now becomes more complex. The set S_0 is no longer a set of improper
organismic sets because a single specialized element e_i does not satisfy Defini-
tion 1d. Thus instead of an aggregate of N organismic improper sets, S_0 now
has a definite structure defined by the set S_R of relations between the different
activities, each of which is now characteristic either of only one element or of
a definite class of elements. If the specialization is complete (I, Def. 5) and if
r_i = N, then the activity of each element maps on the same activity of each
unspecialized element. The relations between the r_i = N different activities
of the N elements are the same as the relations between the r activities of each
unspecialized element. If r_i < N, then we have r_i classes of elements, identical
within each class but different from class to class. The activity of each class
maps on the same activity of the unspecialized element, and the relations
between the different activities are again prescribed.

If the specialization is not complete, still the partly specialized set is more
complex than each improper unspecialized set. The N elements of S_0 are now
divided into v = r classes of a_1, a_2, . . . , a_r elements each, such that the sets
S^{(a)}_{a_i} of activities of different classes are disjoined and that \( \bigcup_{a_i} S^{(a)}_{a_i} = S^{(a)} \). Again
the sets S^{(a)}_{a_i} of activities of each class map on the corresponding subsets of S^{(a)}
of the unspecialized organismic one-element set.

If the unspecialized elements of S_0 are of zeroth order (I, p. 143), that is if
they represent what we suggested calling "protogenes," then the analogy of the
above discussed mappings with those that are obtained through stages T^{(1)}_1,
T^{(1)}_2 and T^{(1)}_3 of the transformation described on p. 353 of (II), or through the
first stage of Figure 2, p. 350 of (II) is evident. The hypothetical "protogene"
plays the role of the primordial graph in the transformation T^{(1)} to T^{(12)} (II,
pp. 353–368). The process of specialization, which results in the evolution of
a more complex organismic set from a simpler one, already induces a relation-invariant mapping of the more complex set on the simpler one. In case of an
incomplete differentiation the set S^{(a)}_c = \bigcap_i S^{(a)}_i (I, p. 145) corresponds to what
we called in (II) the residual biological properties.

However, the mappings studied above are 1–1. We do not have an epi-
morphism. It is, however, introduced to some extent by Postulate 4 of (I)
(p. 146). If to the set S_0 of N elements, a set S_n of partially specialized elements
is added, such that the activities a_{i_1}, a_{i_2}, . . . , a_{i_r} result in the product p_{i_r}, according
to \( A_{i_r} \rightarrow p_{i_r} \), and if that reaction chain replaces a_{i_r} \rightarrow p_{i_r}, then in mapping
the set S_0 \cup S_n on any original unspecialized element of S_0, the activity
a_{i_1}, . . . , a_{i_r} will map on a_{i_r}. Since in general several sets S_{n_1}, S_{n_2}, . . . , S_{n_r} can
thus be added to S_0, the mapping of the activities of the elements of S_0 \cup S_{n_1} \cup
\ldots S_{n_r} on those of the undifferentiated one-element organismic set will be in