WALL SHEAR IN PULSATILE FLOW

A. S. Jones
Department of Mathematics,
University of Queensland,
Brisbane, Queensland

This paper deals with a mathematical attempt to determine the wall shear during normal flow of blood in the ascending and the descending thoracic aorta. A simple model is used, but the results obtained are in agreement with published experimental results for the descending thoracic aorta. It is suggested that the degree of fluctuation in the pressure gradient at a given station is the major factor in determining the level of wall shear at that point.

1. Introduction. Recently, Caro, Fitz-Gerald and Schroter (1969) suggested that early atheroma is coincident with regions in which the shear rate at the arterial wall is locally reduced. Ling, Atabek, Fry, Patel and Janicki (1968) have given crude measurements of wall shear in the descending aorta of a dog, but in the absence of more sophisticated experimental equipment, it is necessary to resort to indirect means to estimate the wall shear. Caro, Fitz-Gerald and Schroter (1970) performed experiments with Evans-blue dye which highlighted some regions of high wall shear in the aorta, but have had to use qualitative arguments to suggest regions of high or low shear. They give three factors which can affect the wall shear: the geometry of the arteries, elasticity of the arterial wall and hydro-dynamic development of the flow. It is with the third of these factors that this paper is concerned.

It has been suggested that wall shear will tend to decrease proceeding downstream from the heart due to spatial development of the boundary layer, in a similar fashion to that observed in steady entry length flow in a pipe.
ever, the profiles calculated for pulsatile flow of the type occurring in the aorta show that effects of this nature are confined to less than one diameter, and that the shear is dependent on the local pulsatility of the flow. Stewartson (1951) showed that for an impulsively started flat plate, the solution is independent of the distance $x$ along the plate if $x > ut$, and Hall (1969) confirmed numerically that the wall shear is independent of $x$ if $x > 0.5ut$. Avula (1969) studied impulsively started laminar flow in the entrance region of a circular tube and obtained a flow field independent of $x$ for $x > 0.4ut$ for small values of $t$. In each case the flow field is independent of $x$ after a distance downstream approximately equivalent to that for which the boundary layer thickness for the steady-state problem is equal to the boundary layer thickness for the unsteady problem with infinite boundaries, and it is assumed that this will be the case in the present problem also. Since the problem of accelerating flow in the entrance of a pipe does not seem to have been attempted, it was necessary to approximate the boundary layer thickness growth at the entrance by using the results corresponding to steady flow with mean velocity equivalent to that occurring in the aorta at the given time, where the boundary layer thickness was defined as the point at which the velocity attains 99% of its mean value. The estimate of one diameter maximum penetration, given above, was obtained in this fashion.

This estimate involves two other assumptions. The first is that the valves can be assumed to be the origin of the boundary-layer growth. This implies that the velocity field at the valves is uniform up to the wall. While this assumption is usually made, it is physically invalid, and, in practice, the origin for the growth of the boundary layer should be placed somewhere in the ventricle. The region of validity of a time-dependent boundary layer could therefore be extended closer to the valves.

The second assumption is that it is only necessary to consider the period of flow during which the pressure gradient in the ascending aorta is negative. This assumption is based on the behavior of boundary-layer flows subjected to adverse pressure gradients and also on the physical behavior of the heart valves. When the pressure gradient becomes positive, the fluid in the boundary layer reverses its direction of motion, forming a new boundary layer at the wall and causing the old boundary layer to separate. The rate of growth of the old boundary layer is consequently of no importance in the estimation of the wall shear. The reversing pressure gradient also causes the valves to close, stemming the entry flow and producing a region of eddy flow in the coronary sinuses. Flow also starts in the coronary arteries, which interacts with the flow in the aorta, encouraging a reversal of the flow near the walls. Consequently, the assumption of a boundary layer developing from the valves into the aorta is no longer valid.