OUTLINE OF A PROBABILISTIC APPROACH TO
ANIMAL SOCIOLOGY: III

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The probabilities of the emergence of the two kinds of social structure in a 3-bird flock (chain and cycle) are deduced under the assumption of certain biases acting on the social dynamics of the flock. In particular a bias against the reversal of peck order and a bias against encounters of individuals of disparate social rank are considered. Likewise a distribution of an "inherent" fighting ability is considered which influences the outcomes of encounters. A functional relation is derived between the importance of this ability and the initial probability of a chain structure.

In previous papers (Rapoport, 1949a,b), hereafter referred to as I and II, we have studied the probability distributions for various types of "social structures" based on a certain asymmetric, but not necessarily transitive, relation between each pair of individuals (peck right). It was pointed out that if peck right depended on a definite quantity of some single variable associated with each individual, then the relation would necessarily be transitive and the only types of social structure would be simple chains of the form

\[ A > B > \ldots Z. \]  

(1)

Structures involving cycles \((A > B > C > A)\) have, however, been observed, and this led us to formulate a number of probabilistic approaches to the theory of the origin of structures, where the outcomes of encounters and thus the emerging of structures is, in part at least, determined by chance events.

In I and II we have examined the distributions of structures emerging from the most "unbiased" kinds of encounters where the probability of victory is always 1/2 for each individual in any encounter. Certainly this is a highly idealized situation. Actually, as has been shown (Collias, 1943) many factors tend to favor victory. Some of these are intrinsic (characteristic of the individual over long periods of time, for example, size of comb); some are accidental (present over short periods, for example, moulting); some are "histori-
In our approach we are interested not so much in the outcomes of encounters between single individuals as in the kinds of social structures that are likely to arise in small flocks. An experimental situation for this approach would be the study of a very large number of small flocks with the view of determining the frequencies with which various types of structure occur initially and in the long run. In the present paper, therefore, we shall inquire how various types of bias influence the probability distribution of those structures.

**Bias against Reversal of Peck Right.**

It was assumed in II that from time to time encounters take place between individuals in a flock and that as a result of such encounters the peck right may be preserved or reversed with equal probability. Under these assumptions, the ultimate, steady state distribution of structures was computed. We shall now introduce a bias against the reversal of peck right, that is, victory is more probable for the dominant individual in any pair. Let us see how this bias affects the ultimate distribution.

The fundamental quantities used in the computation of ultimate distributions are the “social mutation” probabilities, \( a_{ij} \), that is, the probabilities that as a result of some single encounter, structure \( S_i \) shall mutate to structure \( S_j \). Evidently

\[
\begin{align*}
   a_{ij} = & \sum_{k=1}^{l(i,j)} e_k^{(ij)} r_k^{(ij)},
\end{align*}
\]

where \( e_k^{(ij)} \) are the probabilities of those encounters which may affect the mutation \( S_{ij} \), and \( r_k \) are the corresponding probabilities of peck right reversal in those encounters.

In a completely unbiased situation, all the \( e_k^{(ij)} \) are equal to \([\frac{1}{2}N(N-1)]^{-1}\), since all encounters are equally probable; furthermore all the \( r_k \) are equal to \( \frac{1}{3} \). However \( l(i,j) \) varies with the pair \((i,j)\). Thus, for the case of three individuals \( l(1,2) = 1 \), since only one encounter can affect the mutation \( S_{12} \); but \( l(2,1) = 3 \) since every encounter in a cyclic flock can affect \( S_{21} \). Therefore in that case \( a_{12} = 1/6; a_{21} = 1/2 \).

In introducing an anti-reversal bias, we are now modifying the weighting factors \( r_k \). We shall consider first the case of three individuals.

Let \( \delta_1 \leq \frac{1}{3} \) and \( \delta_2 \leq \frac{1}{3} \) be the corresponding biases against the reversal of peck right between two individuals whose social rank differs