A NOTE ON LANDAHL'S THEORY
OF PSYCHOPHYSICAL DISCRIMINATION

N. RASHEVSKY
COMMITTEE ON MATHEMATICAL BIOLOGY
THE UNIVERSITY OF CHICAGO

In his theory of psychophysical discrimination H. D. Landahl assumes that because of the symmetry of the neural circuit considered the fluctuation of the difference of excitations at the two parallel connections is the same as if the fluctuations occurred only at either one of the connections. It is shown that if the excitation at both connections fluctuates independently according to a simple exponential probability function, the difference of excitation fluctuates according to a different probability function.

In his theory of psychophysical discrimination H. D. Landahl (1937) [cf. also Rashevsky, 1948] assumes random fluctuation of excitation in the two parallel cross-inhibited pathways. Because of the symmetry of the whole structure a positive fluctuation on one side is equivalent to a negative fluctuation on the other side. Therefore Landahl considered first fluctuations at only one of the two parallel connections.

In a subsequent paper (Landahl, 1940) it was shown that if the fluctuations are distributed according to the Gaussian law, then the results are the same, regardless of whether one set of fluctuations occurs at one connection or two independent fluctuations occur in both parallel chains.

Inasmuch, however, as a distribution different from the normal has been introduced by H. D. Landahl, and found more convenient for deriving closed expressions, it is interesting to investigate the case of two independent fluctuations for such a distribution.

The purpose of this note is to show that in this case the restriction to one fluctuation is not always justified, although it does not necessarily invalidate Landahl's results. In fact what is essential in Landahl's theory is to assume that the difference,

\[ \Delta = \varepsilon_1 - \varepsilon_2, \]  

fluctuates according to some distribution function \( p(x) \), such that
\[
\int_{-\infty}^{+\infty} p(x) \, dx, \quad (2)
\]

and where \( p(x) \, dx \) denotes the probability of a fluctuation between \( x \) and \( x+dx \). This does not mean, however, as we shall see, that either \( \varepsilon_1 \) or \( \varepsilon_2 \) individually fluctuates according to the same function \( p(x) \).

Let \( \Delta > 0 \) for the sake of definiteness and consider the probability \( P_w \) of the (wrong) reaction \( R_2 \) when both \( \varepsilon_1 \) and \( \varepsilon_2 \) fluctuate independently according to some function \( p(x) \).

Suppose that at a given moment \( \varepsilon_1 \) has a fluctuation \( \phi \), while \( \varepsilon_2 \) has a fluctuation \( \phi' \), so that instead of \( \varepsilon_1 - \varepsilon_2 \) we now have \( (\varepsilon_1 + \phi) - (\varepsilon_2 + \phi') \). Considering for simplicity the case of two categories, that is, \( h = 0 \) (Landahl, 1937; Rashevsky, 1948), we find that for \( R_2 \) to occur we must have \( (\varepsilon_1 + \phi) - (\varepsilon_2 + \phi') < 0 \), or

\[
\phi' > \Delta + \phi. \quad (3)
\]

Hence for a given \( \phi \), any value of \( \phi' \) which satisfies (3) will result in the reaction \( R_2 \).

But, for a fixed \( \phi \), the probability of inequality (3) is given by

\[
\int_{\Delta+\phi}^{\infty} p(x) \, dx = F(\Delta + \phi). \quad (4)
\]

Since the probability of \( \phi \) being between \( \phi \) and \( \phi + d\phi \) is \( p(\phi) \, d\phi \), the probability of \( R_2 \) for a given \( \phi \) is equal to

\[
F(\Delta + \phi) \, p(\phi) \, d\phi. \quad (5)
\]

Since \( \phi \) may have any value from \(-\infty\) to \(+\infty\),

\[
P_w = \int_{-\infty}^{+\infty} F(\Delta + \phi) \, p(\phi) \, d\phi = \int_{-\infty}^{+\infty} p(\phi) \, d\phi \int_{\Delta+\phi}^{\infty} p(x) \, dx. \quad (6)
\]

Let us evaluate \( P_w \) under the assumption made by Landahl that

\[
p(x) = \frac{k}{2} e^{-k|x|}. \quad (7)
\]

Then we have

\[
\text{for } x > 0 \quad p(x) + p_+(x) = \frac{k}{2} e^{-kx}; \quad (8)
\]

\[
\text{for } x < 0 \quad p(x) = p_-(x) = \frac{k}{2} e^{kx}. \quad (9)
\]

For \( \Delta + \phi > 0 \), or for \( \phi > -\Delta \), we find from (4):