COMPARISON OF MATHEMATICAL MODELS FOR CAT LUNG AND VISCOELASTIC BALLOON DERIVED BY LAPLACE TRANSFORM METHODS FROM PRESSURE-VOLUME DATA

J. HILDEBRANDT
Institute of Respiratory Physiology,
Virginia Mason Research Center and Firland Sanatorium,
and the Department of Physiology and Biophysics,
University of Washington, Seattle

The mechanical properties of some hollow organs are most conveniently described by a pressure-volume relationship. If the material exhibits hysteresis, the $p-v$ relation must include provision for time-dependent or path-dependent properties. Provided the amplitude of deformation is fairly small and the hysteresis is primarily of the viscoelastic type, a linear description is possible. That this may take the form of a simple transfer function in which material properties are implicit is illustrated for the case of a rubber balloon. The transfer function was derived from the pressure transients which follow step changes in volume produced in a fluid-filled plethysmograph. The applicability of the transfer function in predicting responses to other forcing functions was tested by varying the balloon volume sinusoidally over a frequency range of 1000, at 4 different amplitudes. The good agreement between the linear model and all types of data justifies the use of Laplace transform methods and the assumption that superposition holds. When isolated cat lung is tested in the same manner, the transfer function quantitatively predicts the magnitude ratio of sinusoidal responses but only about two-thirds of the phase angle. The additional energy loss per cycle is interpreted as arising from static hysteresis. The analysis thus provides a simple means of estimating the relative contributions of viscoelastic (dynamic) and static hysteretic processes to the total damping in a material.

Introduction. Certain mechanical properties of materials can be quantified from data obtained from one-dimensional deformation experiments. However, in the study of soft materials such as biological tissue or elastomers and
polymers it is desirable to have a mathematical model describing two-dimensional deformation as well. The distinction is made because even for isotropic incompressible materials finite biaxial strain cannot in general be calculated from uniaxial experimental data (Green and Adkins, 1960).

One means of producing uniform biaxial deformation in isotropic sheets is by ballooning the sheet through a small orifice, then calculating stress from the pressure and radius of curvature near the center of the bulge (Hildebrandt et al., in press). However, if the material exists as a nearly uniformly inflatable thin-walled structure, such as lung or bladder, or can be moulded into one, as a rubber or synthetic balloon, one may describe biaxial material

\[ p \text{(mm Hg)} \]

\[ v \text{(ml)} \]

Figure 1. Quasi-static \( p-v \) curve of the balloon obtained by stepwise inflation and deflation in 10 ml increments at 1 min intervals. \( p-v \) loops on the deflation limb have an almost linear axis, enabling the application of linear mathematical methods to the analysis. The approximate operating point for subsequent plethysmographic experiments is indicated by (*)

properties by a pressure-volume relationship. Furthermore, in distinction to the orifice technique, the \( p-v \) variables can be monitored continuously making dynamic measurements of viscoelastic characteristics possible. If desired, \( p \) and \( v \) may be converted into stress and strain, or tension and area, provided one knows the geometry.

The full description of a material (constitutive equations) invariably seems to involve nonlinear differential equations. At times, however, the pressure-