CONTRIBUTION TO THE PROBABILISTIC THEORY OF NEURAL NETS: III. SPECIFIC INHIBITION

ANATOL RAPPORT
COMMITTEE ON MATHEMATICAL BIOLOGY
THE UNIVERSITY OF CHICAGO

The input-output formula is derived for a neuron upon which converge the axones of two other neurons (one excitatory, the other inhibitory) which are themselves subjected to a "Poisson shower" of excitatory stimuli. If the period of latent inhibition, \( \sigma \), does not exceed one half the refractory period, \( \delta \), the input-output curve has no maximum. If, however, \( \sigma \geq \delta/2 \), a maximum exists in the input-output curve. As the outside frequency \( x \) increases without bound, the output frequency \( x' \) approaches an asymptotic value which ranges from \( 1/\delta \) to \( 0 \), depending on the ratio \( \sigma/\delta \). The maximum output (if it exists) is also derived as a function of \( \sigma \) and \( \delta \).

In previous papers, I and II of this series (Rapoport, 1950a,b), we derived input-output functions for a single neuron receiving a shower of completely randomized (Poisson-distributed) stimuli. In most examples treated in those papers the stimuli were supposed to come from outside the organism. However, in the case of inhibition, where some of the stimuli were supposed to be excitatory and some inhibitory, it seemed too artificial to suppose that the shower of outside stimuli could be subdivided into two "sub-showers," one excitatory, one inhibitory. We supposed, therefore, that a homogeneous shower of stimuli impinged upon two neurons, and that it was these neurons which produced respectively an excitatory and an inhibitory effect upon a third neuron. The output of the third neuron as a function of the outside input was then derived.

This model was interesting chiefly because of the maximum which characterized the output curve of the third neuron with respect to the outside input. It was shown in I that the existence of such maxima allowed the construction of "filter nets" in which certain neurons would respond only to certain ranges of the outside input frequency. Moreover, these ranges could be narrowed to any desired degree by simply raising the thresholds of the neurons. Thus a counterpart of a resonance phenomenon could be accounted for in a neural net.
The weakness of the filter net model discussed in I lay in the drastic simplification of neglecting the refractory period. This allowed the problem to be treated by quite simple mathematical means. In the present paper, it will be supposed that neurons have a finite, constant refractory period $\delta$. The inclusion of a refractory period complicates the mathematical treatment considerably, as will be seen, since the time distribution of firings of the neurons is no longer a Poisson distribution. Moreover, the existence of a maximum in the output (and hence the possibility of using the model for constructing "filter nets") will now depend on a certain relation between the refractory period and the period of latent inhibition.

Consider a net such as is shown in Figure 1. The firing conditions for such a net in the McCulloch-Pitts notation (McCulloch and Pitts, 1943) would be, under the assumption of quantized time, where the synaptic delay is taken as the unit of time,

$$N_3(t) \leftarrow N_1(t-1) \cdot N_2(t-1).$$

We shall retain the synaptic delay (assumed constant for all neurons) as the unit of time. However, we will not demand that firings occur only at moments which are all multiples of the synaptic delay measured from some common origin, as is supposed in the above-mentioned paper of McCulloch and Pitts. Now if $\delta$ designates the refractory period, and $\sigma$ the period of latent inhibition, we may rewrite equation (1) as follows:

$$N_3(t) \equiv N_1(t-1) \cdot [\tau_1 < \delta \equiv \overline{N_3(t-\tau_1)}] \cdot [\tau_2 < \sigma \equiv \overline{N_3(t-\tau_2-1)}].$$

That is to say, "$N_3$ fires at time $t$, provided $N_1$ has fired at time $(t-1)$, and provided $N_1$ has not fired within the interval $\delta$ before $t$, and provided $N_2$ has not fired within the period of latent inhibition prior to the arrival of the impulse at $N_3$ from $N_1."$

We are interested in the output of $N_3$ as a function of the average frequency $x$ of the outside stimuli. Evidently, the output will not be the same as that derived in I, where the refractory period was neglected. In that case, the stimuli received by $N_3$ from $N_2$ became