THE MECHANISM OF CELL DIVISION

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The recently developed approximation method for treating problems of cell biophysics is generalized and corrected in some points. The equations of elongation of a dividing cell of any shape are given for the most general case of finite permeability as well as of finite internal and external diffusion coefficients.

In the Appendix to our book (Rashevsky, 1938a) we have developed a general approximation method for treating problems of cell deformation under the influence of diffusion. The method while only approximate has the advantage of generality, applying to any elongated shapes without restrictions to an analytically definable boundary surface. At the same time it leads to a very simple and short way of treating such problems, for which an exact mathematical treatment requires a rather involved procedures. H. D. Landahl (1939) has applied this method to the problem of cell respiration and has obtained in a much simpler way the same results which he found previously (Landahl, 1937) by the more elaborate old method. Also A. Weinberg (1939) obtains by this method essentially the same results concerning periodical reactions, as those which are given by the much more elaborate exact method (Weinberg, 1938).

A comparison of the equations for the rate of elongation of a dividing cell, obtained in the Appendix (Rashevsky 1938a), to those obtained in a particular case exactly has been made by G. Young (1939), who worked out the exact expression for an ellipsoid of revolution. This comparison again shows the applicability of the approximate method. The differences found are of no practical consequence, since no dividing cell has a regular shape, and the application to such a cell of formulae which hold exactly for an ellipsoid only would be likely to involve as large an error as that by which the approximate equations differ from the exact ones.

In the original exposition of the method given in the Appendix to our book (1938a) we have for sake of simplicity considered the elongation of the cell only in a very special case, namely when both the permeability $h$ and the external diffusion coefficient $D_e$ are infinite, although the diffusion problem was treated for the more general case of finite $h$ and $D_i$ (Rashevsky, 1938a, p. 317). As has been shown by G. Young (1939), in the case of $h = D_e = \infty$ however, we
should obtain a zero elongation, since \( c \) is constant along the surface. The error committed by us lies in the discussion on p. 308 (Rashevsky, 1938a), where the force appears as a small difference of two quantities. The use of Betti’s theorem (Love, 1906, p. 172; Young, 1939), combined with our approximate method for determining the distribution of concentrations, leads to a much more straightforward derivation of the elongation equation and avoids the above mentioned doubtful argument on p. 308 (Rashevsky, 1938a).

Betti’s theorem, applied to a plastic body (Young, 1939), gives for the average rate of elongation of a body of any shape of volume \( V \) in the direction of the \( z \)-axis:

\[
\frac{1}{a} \frac{da}{dt} = \frac{1}{3 \eta V} \int \int \int [z \frac{Z}{3V} - \frac{1}{2} \{y \frac{Y}{V} + x \frac{X}{V}\}] \, dV + \frac{1}{3 \eta V} \int \int [Z \frac{Z}{V} - \frac{1}{2} \{X \frac{X}{V} + Y \frac{Y}{V}\}] \, dS
\]

where \( X, Y \) and \( Z \) are the components of the force per unit volume and \( X_v, Y_v \) and \( Z_v \) are the components of any surface force. The triple integral is extended over the whole volume of the body, the double integral over the surface.

If \( X, Y, Z \) are derived from a potential, so that (Rashevsky, 1938a, Chapt. VII, VIII, IX):

\[
F = -\frac{RT\mu}{M} \text{grad} \, c
\]

then as is shown by G. Young (1939), the volume integral transforms into a surface integral of the form:

\[
-\frac{RT\mu}{M} \int \int [z \cos(v, z) - \frac{1}{2} \{x \cos(vx) + y \cos(vy)\}] \, dS.
\]

Therefore we have for the average rate of elongation for a body of any shape exactly:

\[
\frac{1}{a} \frac{da}{dt} = -\frac{RT}{3M\eta V} \mu J + \frac{1}{3\eta V} T \tag{1}
\]

where, taking \( z \) as the direction of elongation:

\[
J = \int \int c \{z \cos(vz) - \frac{1}{2} \{x \cos(vx) + y \cos(vy)\}\} \, dS \tag{2}
\]

and

\[
T = \int \int [Z_v z - \frac{1}{2} \{X_v x + Y_v y\}] \, dS \tag{3}
\]

\( X_v, Y_v, Z_v \) being the components of the force \( \frac{RT}{M} \) \((c_i - c_e)\) at the membrane. For \( h = D_e = \infty \), \((c_e - c_i) = 0\) and \( c \) is constant along...