NOTE ON THE HAMILTONIAN PRINCIPLE IN BIOLOGY AND IN PHYSICS

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A suggestion is made to establish a general variational principle of which the recently discussed principle of maximum energy exchange in biology and the usual Hamiltonian principle in physics would be particular cases.

In a previous publication (Rashevsky, 1943), we have formulated mathematically a hypothesis stated some time ago by Alfred J. Lotka (1922 a, b), namely, that the course of events in the biological world is determined by maximizing the total energy exchange between different biological units. We have suggested that this principle may be stated in form of a variational principle. If \( x_i \)'s are any kind of biological quantities that determine the energy exchanges in a biological system, and \( \dot{x}_i \)'s are the corresponding derivatives with respect to time, we construct a "Hamiltonian" \( F(x_1, \ldots, x_n, \dot{x}_1, \ldots, \dot{x}_r) \), and require

\[
\delta \int_t^{t+\delta} F \, dt = 0,
\]

so as to maximize the integral. Numerous possible applications of that principle to a great variety of biological problems have been discussed in loc. cit.

In physics, the "Hamiltonian" \( H \) is a function of the generalized coordinates \( q \) and momenta \( p \), and we have

\[
\delta \int_t^{t+\delta} H \, dt = 0,
\]

minimizing the integral.

The question naturally arises as to whether principle (1) may not be reduced to (2), so as to avoid a duality of living and non-living. This does not seem likely, and A. J. Lotka (1922 a, b) has proposed his hypothesis as an independent principle. It must be remarked that \( F \) has the dimension of energy flow, hence \( [F] = ml^2 t^{-3} \), while \( H \) has the dimension of energy, and therefore \( [H] = ml^2 t^2 \). One might per-
haps think that in some way both principles operate simultaneously, both in the living and non-living, but that the role of the first principle is more pronounced in the living world, that of the second in the non-living. The most natural suggestion would be to put

$$\delta \int_{t_1}^{t_2} (\alpha_1 F - \alpha_2 H) dt = 0,$$

and assume that the first term prevails in the living, the second in the non-living. If, however, we make such an assumption, we must assume $\alpha_1$ and $\alpha_2$ to be of different physical dimensionality, so as to make $\alpha_1 F$ and $\alpha_2 H$ of the same dimensionality. This introduces a rather unpleasant indefiniteness.

It may be suggested that $\alpha_1$ and $\alpha_2$ be chosen so as to make the whole integral in (3) a pure number. This would give

$$[\alpha_1] = m^{-1} l^2 t^2; \ [\alpha_2] = m^{-1} l^2 t .$$

We may make the hypothesis that $\alpha_1$ and $\alpha_2$ are two universal constants. All systems for which $\alpha_1 F >> \alpha_2 H$ we may define as biological; all systems for which $\alpha_1 F << \alpha_2 H$ we may define as physical. For each of those classes both $\alpha_1$ and $\alpha_2$ drop out of the picture. But when $\alpha_1 F$ is comparable to $\alpha_2 H$, we have intermediate systems in which $\alpha_1$ and $\alpha_2$ play a role.

However, inasmuch as mathematically only the ratio $\tau = \alpha_1/\alpha_2$ is important in the variational problem given by equation (3), therefore it may be better to leave the integral of the dimension energy $\times$ time by putting

$$\delta \int_{t_1}^{t_2} (\tau F - H) dt = 0 .$$

We may assume $\tau$, which has the dimension of time, to be a universal constant characteristic of biological phenomena. If $\tau$ is very small, then systems in which energy exchanges are not too large fall into the domain of physics. Systems in which the energy exchanges play a preponderant role fall in the domain of biology. When $\tau F$ is comparable to $H$ we have borderline systems. Such may be perhaps the case of single cells. By developing a theory of such systems, we may determine the constant $\tau$ from any comparison of quantitative data with the theory.

If principle (5) is to hold quite generally, then it must also describe the process of the beginning of life from the non-living. Since in this process we must obviously have a borderline case, therefore we shall have to consider the general expression (5) in which $\tau F$ and