MATHEMATICAL BIOPHYSICS OF SOME NEURAL NETS

J. B. ROBERTS

THE UNIVERSITY OF CHICAGO

Methods are considered for constructing neural nets of the McCulloch-Pitts type giving arbitrary time delays between stimuli and their responses. By introducing cycles of neurons, the problem is treated from the standpoint of economizing on the number of neurons required. A generalization is effected that will realize any unique temporal response pattern from a unique stimulus.

Time Delay Circuits. The "microscopic picture" of neural nets (McCulloch and Pitts, 1943) considers individual neurons and synapses. Time is regarded as "quantized" rather than continuous, and only spatial and not temporal addition of impulses at synapses is provided for.

The construction of a neural net such that any desired finite time delay, of $N$ units between the action of a stimulus $S$ and the action of a response $R$, is incorporated in the net, can be made very simply by inserting a chain of $N-1$ synapses between the neuron stimulated and that responding. Clearly, we can make the delay as long as we desire.

However, we wish to devise a net that will give a time delay of, say, $N$ time units using less than $N$ synapses. Such an economy can be realized in neural nets involving cycles of neurons, i.e., sets of neurons $b_1, \ldots, b_r$ connected in "series" such that $b_r$ is connected to $b_1$. Thus a cycle is a repeating system.

By using such cycles we can devise, with relatively few neurons, nets with time delays of any finite length. In the example shown in Figure 1, we have two cycles, of five synapses and of seven synapses respectively. The response $R$ will occur at 36 time units after $S$. The net represented in Figure 1 gives this time delay of 36 with the use of only 13 synapses. The dotted lines in the Figure represent connec-
tions of other neurons to response $R_1$, which occurs at 19 time units after $S$. In general, this net will give a response for any time delay from 2 to 36. Any such response for this net will be repeated at intervals of 35 time units. Note that the period $P$ of the net is equal to the product of the numbers of synapses in the individual cycles and that the “range” of time delays this circuit can give is from 2 to $P + 1$.

We will now discuss the specific construction of a net for some particular time delay $O_r$. We will represent the number of cycles by $r$ and the number of synapses in each cycle by $N_i (i = 1, \ldots, r)$.

In the construction of the nets under consideration we will use the following two notions:

(1) The effective factors (EF’s) of a net (defined for nets of the type shown in Figure 1 only). For these nets there is one EF for each cycle, and the EF corresponding to any particular cycle has a value equal to the number of synapses in that cycle.

(2) The highest common normal factor of a set of $r$ numbers ($HCNF_r$). This is defined by the following relation:

$$HCNF_r = \frac{\prod_{i=1}^{r} N_i}{LCM(N_1, \ldots, N_r)},$$

where $LCM$ is the “lowest common multiple.”

It can be verified that

$$LCM(N_1, \ldots, N_r) = P(N_1, \ldots, N_r),$$

where $P$ is the period of the net. For a circuit similar to that of Figure 1 we know that the range is from 2 to $P + 1$ and therefore also from 2 to $LCM(N_1, \ldots, N_r) + 1$. So, given an $O_r$ value, we can state the following. If

$$LCM(N_1, \ldots, N_r) + 1 \geq O_r,$$

then the net with cycles $N_1, \ldots, N_r$ will realize a time delay of $O_r$ time units.

Combining equations (1) and (3) we get

$$\frac{\prod_{i=1}^{r} N_i}{HCNF_r} \geq O_r - 1.$$

It will now be shown that the $HCNF_r$ of $N_1, \ldots, N_r$ is equal to the $HCF$ (highest common factor) of the $r$ products.