A CASE OF APPARENT ACTIVE TRANSPORT. I.

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When an experimenter determines the “internal concentration” of a substance in a cell (or cell suspension) it is in general the average concentration (quantity of substance divided by cell volume or volume of cell water) which is measured. When this concentration is less than that in the ambient medium but there is either no flow into the cell or flow from the cell into the medium, then (under the usually tacit assumption of spatial uniformity in the cell) the possibility of active transport is considered. The possibility that lack of spatial uniformity could lead to apparent active transport was early proposed by A. Bierman and later examined quantitatively by N. Rashevsky for a special case. In this paper spherical cells are treated but under quite general conditions regarding the metabolic aspects of the problem. It is shown that apparent active transport can result for a metabolite which is a reactant in one set of reactions and a product in another provided the sites of these sets of reactions are spatially separated in the cell.

Some years ago, A. Bierman (1953) proposed the following interesting possibility. Suppose a cell consists of two concentric regions such that in the central or inner region a metabolite is consumed while in the peripheral region it is produced. If in the steady state there is net production for the whole cell, diffusion will occur from the outer region into the surrounding medium. Given this, is it possible for the rate of consumption in the central region to be sufficiently great that the average concentration for the whole cell is less than the concentration in the medium?

If so, then this situation would appear to constitute a case of active transport. For, in comparing the intracellular and extracellular concentrations, it is in general operationally necessary to compare the mean or average cellular
concentration and the external concentration. In other words, in terms of the observable mean cellular concentration, the metabolite is "observed" to flow from a region of low to a region of high concentration, although in terms of the detailed spatial distribution of concentration (usually inaccessible to the experimenter) the laws of diffusion are strictly obeyed at every point of the system.

The question is not trivial since a rate of consumption sufficiently great to render the mean cellular concentration less than the external concentration may also be sufficient to lead to net consumption for the entire cell, in which case the flow of substance between cell and medium is in the expected direction. The outcome of these two effects of the rate of consumption in the inner region is not obvious and requires some detailed analysis.

Actually Bierman (1953), having posed the question in the first paragraph above, proceeded to solve a different problem.* Rashevsky (1960) did consider a particular case and he showed that subject to certain simplifying assumptions it is indeed possible to have flow from the cell while the mean cellular concentration is less than the external concentration.

In what follows we consider, as did Rashevsky, only the case of a spherical cell and denote by $Q$ the rate of production at any point in the cell and by $\bar{Q}$ the mean (or volume average) of $Q$. At any point where consumption occurs $Q < 0$. When consumption occurs in the central sphere and production in the concentric spherical shell we write $Q = (-, +)$ and in the reverse situation $Q = (+, -)$. The difference $\bar{C} - C^0$ between the mean cellular concentration and the external concentration is written as $\Delta = \bar{C} - C^0$. For $Q = (-, +)$ it is shown that from $\bar{Q} = 0$ it necessarily follows that $\Delta < 0$, and that there exists a range of values of $\bar{Q} > 0$ such that $\Delta < 0$ but that $\bar{Q} < 0$ with $\Delta > 0$ is impossible. Similar results are established for $Q = (+, -)$. These conclusions follow rather directly from a general formulation for $\bar{C}$ (Hearon, 1953b), and under the mildest restrictions upon the functional form of $Q$.

Consider a spherical cell of radius $r_o$ in which a given metabolite is produced at the rate (per unit volume) $Q$. We consider only spherical symmetry and $Q$ is a function of $r$ alone but we admit that this may result from the fact that $Q$ is a function of the concentration of the metabolite and of $r$, $Q(r) = q(C, r)$,

* Bierman makes the assumption that what is experimentally determined is the sum of two species governed by a conservation condition. For example if $A$ and $B$ are interconverted mole for mole, it is $\bar{C}_A + \bar{C}_B$ which is taken as the mean observed concentration. As it turns out, when such a sum (in general linear combination) of concentrations is the procedural variable an interesting possibility of apparent active transport arises. However, the existence and distribution of regions of consumption and production are then irrelevant to the problem. The two problems (distributed regions and determination of linear combinations) are best treated separately. In a subsequent paper (Hearon, 1965) the latter of these problems is considered in some detail.