ON THE LAWS OF PSYCHOPHYSICS

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Experiments by S. S. Stevens (Stevens, 1957, and Stevens and Galanter, 1957) and his collaborators indicate that the so-called logarithmic Weber-Fechner Law is not realized in most human perceptions. Instead, a power law seems to emerge over a large number of sensory continua. This is important because for a long time the logarithmic law was looked upon as almost the only possible psychophysical law. The logarithmic law appeared desirable intuitively because it made the sensation depend on the relative values of the stimuli and not on their absolute values. This is, of course, useful for evolutionary reasons. Some other reasons are also discussed by Stevens (1961).

In the following we analyze the evolutionary ideas involved in the case of brightness perception of the human eye. It will be seen that power laws are also consistent with evolutionary requirements. A more extensive discussion covering other examples in audio, tactile, and taste perceptions will be published elsewhere.

Consider a scene illuminated with an illuminant of intensity, \( I \), in which various objects \( O_1, O_2, O_3, \ldots \) are perceived with brightnesses \( S_1, S_2, S_3, \ldots \). What will happen to these brightness sensations if the illuminant is changed to a different level, \( I' \)?

The answer to this question is not at all easy. It involves several assumptions cutting deeply into the nature of our perceptual make-up and the evolutionary requirements of maximum survival value. The following general assumptions are made:

(A) Human perceptual apparatus works as a comparison device. It does not assign sensations \( S_1, S_2, S_3, \ldots \) to \( O_1, O_2, O_3, \ldots \) in the absolute sense. Rather,
the perceptions are geared to relationships between \((O_1O_2), (O_2O_3), (O_1O_3)\ldots\) as \(S_{12}, S_{23}, S_{13}\ldots\).

(B) Evolution requires that as the illuminants are changed to different levels \(I', I'', \ldots\) the associated perceptions remain invariant.

From these two assumptions it can be deduced that when the intensity changes to a level \(I' = \kappa I\), the relation between any two perceptions should satisfy the relationship

\[
S_{12}(i_1, i_2) = S_{12}(\kappa i_1, \kappa i_2),
\]

since \(i'_1 = \kappa i_1, i'_2 = \kappa i_2, \ldots\) etc. Here \(i_1\) and \(i_2\) are the light intensities that reach the eye from the two objects \(O_1\) and \(O_2\). Consequently we have the general formula

\[
S_{12}(i_1, i_2) = F\left(\frac{i_2}{i_1}\right).
\]

It is important to note at this point that no further conclusions can be derived unless we make specific assumptions as to the combination law of sensations operating in the brain. In fact, we will show now that to each mechanism of combination there corresponds a new stimulus-sensation law. We will give two explicit examples below:

(A) Let the sensation mechanism in the brain produce additive responses, namely, \(S_{12} = S_2 - S_1\). Putting \(dS = S_2 - S_1 = S + dS - S\), and dropping the indices we have

\[
dS = F\left(\frac{i + di}{i}\right) = F\left(1 + \frac{di}{i}\right).
\]

Then by expanding into the Taylor series we find

\[
dS = F'(1) \frac{di}{i} + \cdots
\]

From this we obtain in the first order

\[
F(1) = 0, \quad F'(1) = \alpha, \quad dS = \alpha \frac{di}{i}
\]

which results in the logarithmic Weber-Fechner Law

\[
S_{12} = \alpha \log \left(\frac{i_2}{i_1}\right).
\]

(B) If we assume the combination of sensations to be a ratio (that is, if