A general theory for the design of apparatus for the low temperature drying of frozen tissues has been developed. Using this theory it is possible to compute drying rates for different materials in different apparatuses and to design an apparatus suitable for the required purpose. It is shown that for the drying of animal tissues in the laboratory, where the area is small, the requirements of the apparatus are not particularly critical. The same theory applied to the drying of biologicals and plasma indicates that the design is exceedingly critical and suggests directions in which existing apparatus might be modified.

Introduction. In a preceding paper the general theory of the vacuum drying of frozen tissues was developed (Stephenson, 1953). In this paper the theory will be applied to the design of drying apparatus.

Efficiency of drying apparatus.† We will define the efficiency of a drying apparatus as the ratio of the minimum possible drying time to the actual drying time. We have the general relation

$$ w \frac{dx}{dt} = \frac{dm}{dt} = \frac{K}{L} (T_e - T_s) = \frac{fP_s}{(2\pi RT_e)^{1/2}}, $$

where

- $w$ is the water content of the frozen tissue or solid,
- $dx/dt$ is the rate of recession of the interface separating the dried and undried portions,
- $dm/dt$ is the rate of evaporation per unit area,
- $K$ is the general coefficient of net heat transfer to the evaporating surface, per unit area,
- $L$ is the latent heat of evaporation of ice,
- $T_e$ is the average absolute temperature of the surrounding environment,
- $T_s$ is the absolute temperature of the subliming interface,

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† Equations (1), (2), and (3) are derived in the preceding paper (Stephenson, loc. cit.). The factor $a$ has been omitted because it is dimensionless and equals unity.
$P_e$ is the saturated vapor pressure of ice at temperature $T_e$, $R$ is the gas constant per gram of water vapor, and $f$ is the probability that an evaporated water molecule is removed from the system before it recondenses on the interface. All units are in the c.g.s. system.

Also

$$f = \frac{1}{f_s + \frac{1}{f_e} - 1},$$

where

$f_s$ is the probability that an evaporated water molecule reaches the exterior surface of the tissue before it is recaptured, and

$f_e$ is the probability that a water molecule which reaches the exterior surface is removed from the system before it re-enters the dried portion of the tissue.

From (1) we have for the time required to dry a thickness $x$,

$$t = \frac{w}{f_e} \int_0^{x} \frac{L}{K(T_e - T)} \, dx = \frac{w}{f_e} \int_0^{x} \frac{(2\pi RT_e)^{1/2}}{f_P} \, dx.$$  \hspace{1cm} (3)

As has been shown in the preceding paper, $f_s$, $f_e$, and $K/L$ can be determined as functions of the thickness of the dry shell and (3) can be integrated. In order to obtain the minimum drying time, we assume that the temperature drop is negligible and that $f_e$ equals one. Then

$$t_{\text{min}} = \frac{(2\pi RT_e)^{1/2}}{P_e} \int_0^{x} \frac{dx}{f_s}.$$  \hspace{1cm} (4)

where $P_e$ is the saturated vapor pressure at temperature $T_e$.

According to our definition, in order for a drying apparatus to have a good efficiency two conditions must be simultaneously satisfied

$$T_e \approx T_c$$  \hspace{1cm} (5)

and

$$\frac{f_s}{f_e} \ll 1 \quad \text{or} \quad f_e \approx 1.$$  \hspace{1cm} (6)

Under some conditions it may be impossible to satisfy (5) and hence theoretically it would be better to define $t_{\text{min}}$ as the drying time when $f_e$ equals one and $K$ is a theoretical maximum. However, this definition would be useless unless the theoretical maximum of $K$ could be determined.

When (5) and (6) are satisfied we have

$$\text{efficiency} \approx 1 - \frac{f_s}{f_e} \approx \frac{f}{f_s}$$  \hspace{1cm} (7)

and, since

$$P_e f_s \approx P_e f_e = P_e f,$$  \hspace{1cm} (8)