BLOCK PLAN FOR A FRACTIONAL 2\textsuperscript{m} FACTORIAL DESIGN
DERIVED FROM A 2\textsuperscript{r} FACTORIAL DESIGN

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Summary

For a given fractional 2\textsuperscript{m} factorial (2\textsuperscript{m}-FF) design T, the constitution of a block plan to divide T into k (2\textsuperscript{r-1} < k \leq 2\textsuperscript{r}) blocks with r block factors each at two levels is proposed and investigated. The well-known three norms of the confounding matrix are used as measures for determining a “good” block plan. Some theorems concerning the constitution of a block plan are derived for a 2\textsuperscript{m}-FF design of odd or even resolution. Two norms which may be preferred over the other norm are slightly modified. For each value of N assemblies with 11 \leq N \leq 26, optimum block plans for k=2 blocks with block sizes \lfloor N/2 \rfloor and N-\lfloor N/2 \rfloor minimizing the two norms are presented for A-optimal balanced 2\textsuperscript{m}-FF designs of resolution V given by Srivastava and Chopra (Technometrics, 13, 257-269).

1. Introduction

Consider a 2\textsuperscript{m} factorial experiment with m factors. An assembly (or treatment combination) is represented by an m-rowed vector (j\textsubscript{1}, j\textsubscript{2}, ..., j\textsubscript{m}), where j\textsubscript{t} (level of t\textsuperscript{th} factor) is equal to 0 or 1. As unknown effects, \theta\textsubscript{0}, \theta\textsubscript{t}, and in general, \theta\textsubscript{t_1...t_k} denote the general mean, main effect of t\textsuperscript{th} factor, and k-factor interaction of t\textsubscript{1}, ..., t\textsubscript{k} factors, respectively. For a fixed integer l (1 \leq l \leq m), let \theta be the \nu \times 1 vector composed of effects up to l-factor interactions, where \nu = \sum_{i=0}^{l} \binom{m}{i}, i.e.,

\theta' = (\theta\textsubscript{0}; \theta\textsubscript{1}; \theta\textsubscript{1,1}; \theta\textsubscript{1,2}; ..., \theta\textsubscript{m;1}; \theta\textsubscript{12,...,i}; ..., \theta\textsubscript{m-1,m}; \theta\textsubscript{12,...,i,...,m}).

Assume throughout this paper that (l+1)-factor and higher order interactions are negligible and that the m factors are different from block factors. Let T be a fractional 2\textsuperscript{m} factorial (2\textsuperscript{m}-FF) design which is a

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suitable set of $N$ assemblies. Note that the assemblies in $T$ are not always distinct. Using a design $T$, we consider the estimation of a $\nu \times 1$ vector of linear parametric functions $\theta_0 = C \theta$ for some $\nu \times \nu$ matrix $C$. For an $N \times 1$ observation vector $y_T$ of $T$ (whose observations are assumed to be independent random variables with common variance $\sigma^2$), consider the model

$$E(y_T) = E \theta$$

where $E(\cdot)$ stands for an expected value and $E$ is the $N \times \nu$ design matrix with elements $\pm 1$ (see, e.g., Yamamoto, Shirakura and Kuwada [13]). Suppose that there exists a $\nu \times \nu$ matrix $K$ satisfying $KM = C$ (i.e., rank $M = \text{rank}[M: C']$), which is equivalent to the estimability of $\theta_0$, where $M = E'E$ is called the information matrix of $T$. The best linear unbiased estimate of $\theta_0$ can then be given by

$$\hat{\theta}_0 = KE'y_T.$$  

When $\theta_0 = \theta$, i.e., $C = I$ (identity matrix of appropriate order) and $\nu_0 = \nu$, $T$ corresponds to a $2^m$-FF design of resolution $2^{l+1}$. In this case, note that the nonsingularity of $M$ is assumed. On the other hand, when $\theta_0 = (\theta_1, \ldots, \theta_m ; \ldots; \theta_{12}, \ldots; \ldots, \theta_{m-1+2\ldots m})'$, i.e., $C = [0 : I : O]$ ($0$ and $O$ are respectively zero vector and zero matrix of appropriate orders) and $\nu_0 = \nu - 1 - \binom{m}{l}$, $T$ corresponds to a design of resolution $2^l$ (see Box and Hunter [1]).

In order to get $\hat{\theta}_0$ in (1.2), it is required to make the plots of $N$ assemblies under conditions as homogeneous as possible for the $m$ factors. After planning a design $T$ for $\hat{\theta}_0$, however, it may occur that $N$ observations for $T$ can not be yielded simultaneously by physical, chemical and/or economical reasons, etc. For example, consider an experiment of a certain reaction for a mixture of $m$ raw materials each at two levels. Then, after accommodating the $N$ mixtures in a given $T$, it may occur that each reaction of them can not be observed under a homogeneous condition. Therefore we consider an arrangement of $T$ in some blocks. The less is the number of assemblies in which we have to experiment simultaneously, the larger is the possibility that we obtain a homogeneous condition. Of course, the number of blocks (say $k$) should be small compared to $N$. The problem is to constitute the $k$ blocks such that the estimate $\hat{\theta}_0$ is not sensitive to the block division. The present paper discusses this problem under special situations. For the $k$ blocks, we use a $2^r$ design with $r$ factors each at two levels which consists of $k$ distinct assemblies ($2^{r-1} < k \leq 2^r$). As block factors, for example, consider experimenter, day and place. Our situation is then the case where the $N$ observations in a given $T$ have to be yielded