BLOCK PLAN FOR A FRACTIONAL 2^r FACTORIAL DESIGN
DERIVED FROM A 2^s FACTORIAL DESIGN

TERUHIRO SHIRAKURA

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Summary

For a given fractional 2^r factorial (2^-FF) design T, the constitution of a block plan to divide T into k (2^r-1 < k ≤ 2^r) blocks with r block factors each at two levels is proposed and investigated. The well-known three norms of the confounding matrix are used as measures for determining a "good" block plan. Some theorems concerning the constitution of a block plan are derived for a 2^r-FF design of odd or even resolution. Two norms which may be preferred over the other norm are slightly modified. For each value of N assemblies with 11 ≤ N ≤ 26, optimum block plans for k = 2 blocks with block sizes \([N/2]\) and \(N - [N/2]\) minimizing the two norms are presented for A-optimal balanced 2^-FF designs of resolution V given by Srivastava and Chopra (Technometrics, 13, 257-269).

1. Introduction

Consider a 2^m factorial experiment with m factors. An assembly (or treatment combination) is represented by an m-rowed vector \((j_1, j_2, \ldots, j_m)\), where \(j_t\) (level of \(t\)th factor) is equal to 0 or 1. As unknown effects, \(\theta_0, \theta_t, \) and in general, \(\theta_{t_1 \cdots t_k}\) denote the general mean, main effect of \(t\)th factor, and \(k\)-factor interaction of \(t_1, \ldots, t_k\)th factors, respectively. For a fixed integer \(l \ (1 \leq l \leq m)\), let \(\theta\) be the \(\nu \times 1\) vector composed of effects up to \(l\)-factor interactions, where \(\nu = \sum_{i=0}^{l} \binom{m}{i}\), i.e.,

\[\theta' = (\theta_0; \theta_1; \theta_2; \cdots; \theta_m; \theta_{12}; \cdots; \theta_{m-1 \ m}; \theta_{12 \cdots l}; \cdots; \theta_{m-l+1 \cdots m}).\]

Assume throughout this paper that \((l+1)\)-factor and higher order interactions are negligible and that the \(m\) factors are different from block factors. Let T be a fractional 2^m factorial (2^-FF) design which is a

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suitable set of $N$ assemblies. Note that the assemblies in $T$ are not always distinct. Using a design $T$, we consider the estimation of a $\nu \times 1$ vector of linear parametric functions $\theta_0 = C\theta$ for some $\nu \times \nu$ matrix $C$. For an $N \times 1$ observation vector $y_T$ of $T$ (whose observations are assumed to be independent random variables with common variance $\sigma^2$), consider the model

$$E(y_T) = E\theta$$

where $E(\cdot)$ stands for an expected value and $E$ is the $N \times \nu$ design matrix with elements $\pm 1$ (see, e.g., Yamamoto, Shirakura and Kuwada [13]). Suppose that there exists a $\nu \times \nu$ matrix $K$ satisfying $KM = C$ (i.e., $\text{rank } M = \text{rank } [M:C']$), which is equivalent to the estimability of $\theta_0$, where $M = E'E$ is called the information matrix of $T$. The best linear unbiased estimate of $\theta_0$ can then be given by

$$\hat{\theta}_0 = KE'y_T.$$  

When $\theta_0 = \theta$, i.e., $C = I$ (identity matrix of appropriate order) and $\nu_0 = \nu$, $T$ corresponds to a $2^m$-FF design of resolution $2l + 1$. In this case, note that the nonsingularity of $M$ is assumed. On the other hand, when $\theta_0 = (\theta_1, \cdots, \theta_m; \cdots; \theta_{12:\cdots:\nu}, \cdots, \theta_{m-1+2:\cdots:m})'$, i.e., $C = [0 : I : O]$ ($0$ and $O$ are respectively zero vector and zero matrix of appropriate orders) and $\nu_0 = \nu - 1 - \binom{m}{l}$, $T$ corresponds to a design of resolution $2l$ (see Box and Hunter [1]).

In order to get $\hat{\theta}_0$ in (1.2), it is required to make the plots of $N$ assemblies under conditions as homogeneous as possible for the $m$ factors. After planning a design $T$ for $\hat{\theta}_0$, however, it may occur that $N$ observations for $T$ can not be yielded simultaneously by physical, chemical and/or economical reasons, etc. For example, consider an experiment of a certain reaction for a mixture of $m$ raw materials each at two levels. Then, after accommodating the $N$ mixtures in a given $T$, it may occur that each reaction of them can not be observed under a homogeneous condition. Therefore we consider an arrangement of $T$ in some blocks. The less is the number of assemblies in which we have to experiment simultaneously, the larger is the possibility that we obtain a homogeneous condition. Of course, the number of blocks (say $k$) should be small compared to $N$. The problem is to constitute the $k$ blocks such that the estimate $\hat{\theta}_0$ is not sensitive to the block division. The present paper discusses this problem under special situations. For the $k$ blocks, we use a $2^r$ design with $r$ factors each at two levels which consists of $k$ distinct assemblies ($2^{r-1} < k \leq 2^r$). As block factors, for example, consider experimenter, day and place. Our situation is then the case where the $N$ observations in a given $T$ have to be yielded.