IMPROVEMENTS ON THE ARC-LENGTH-TYPE METHOD

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ABSTRACT: Arc-length-type and energy-type methods are two main strategies used in structural nonlinear tracing analysis, but the former is widely used due to the explicitness and clarity in conception, as well as the convenience and reliability in calculation. It is very important to trace the complete load-deflection path in order to know comprehensively the characteristics of structures subjected to loads. Unfortunately, the nonlinear analysis techniques are only workable for tracing the limit-point-type equilibrium path. For the bifurcation-point-type path, most of them cannot secure a satisfactory result. In this paper, main arc-length-type methods are reviewed and compared, and the possible reasons of failures in tracing analysis are briefly discussed. Some improvements are proposed, a displacement perturbation method and a force perturbation method are presented for tracing the bifurcation-point-type paths. Finally, two examples are analyzed to verify the ideas, and some conclusions are drawn with respect to the arc-length-type methods.

KEY WORDS: arc-length-type methods, limit point, bifurcation point, displacement perturbation method, force perturbation method

1 INTRODUCTION

In the last decades, with the wide use of various large-span space structural systems, significant progress has been made in the area of nonlinear analysis techniques[1~12]. It is known that for large-span space structural systems, the limit-point type instability and bifurcation-point type instability are two main types of instability with different characteristics. The former concerns a kind of geometric softening behavior under external loads with unique equilibrium path, while the latter is just a shift of equilibrium states among several possible paths. The instability modes of large-span space structures are very complex and usually will include both the limit-point type and the bifurcation-point type instability. In order to know comprehensively the characteristics of the systems, tracing the complete load-deflection path is very important. Arc-length-type methods have been widely used to trace the complete load-deflection paths due to the explicitness and clarity in conception, as well as the convenience and reliability in calculation. Unfortunately, the existing techniques cannot give satisfactory results for bifurcation paths. In the paper, main arc-length type methods are reviewed and compared, and the most possible reasons of failures in tracing particular paths are briefly discussed. Based on the analysis of possible instability modes, a displacement perturbation method and a force perturbation method are proposed to overcome the problems.

2 IMPROVEMENTS ON ARC-LENGTH-TYPE METHODS

2.1 Review on Arc-length-type Methods

Generally, incremental equations in structural nonlinear static analysis take the following form

$$K_t \Delta q = \Delta \lambda P + R$$  \hspace{1cm} (1)

where $K_t$ is the current tangent stiffness matrix, $\Delta q$ is the displacement increment vector, $\Delta \lambda$ is the loading increment parameter, $P$ is the external load reference vector and $R$ is the residual force vector.

The unknown variables in Eq.(1) are $\Delta q$ and
$\Delta \lambda$, with a total number of $N + 1$. But the number of the equations is just $N$. So one constraint equation as follows is necessary

$$C(\Delta q, \Delta \lambda) = 0$$

(2)

The names of arc-length methods are derived from the type of the constraint equation they used. Each method has four essential parts: (1) controlling the arc increment; (2) controlling the load increment parameter (the constraint equation); (3) determining the sign of the initial loading increment parameter; (4) solving the loading increment parameter.

In the $j$-th iterative step of the $i$-th incremental step, $\Delta q^i_j$ is the total displacement increment, $\delta q^i_j$ is the total loading increment parameter, $\Delta \lambda^j_i$ is the total loading increment parameter, and $\delta \lambda^j_i$ is the loading increment parameter corresponding to the residual force. The geometrical interpretation of each variable from load step $i$ to $i + 1$ can be shown in Fig.1. For clarity, the subscript of the $i$-th incremental step in Fig.1 was omitted. The vectors $(\Delta t^j)_i$, $(\delta t^j)_i$ and $(\delta t^j)_i$ are defined as follows

$$\begin{align*}
(\Delta t^j)_i &= (\Delta q^j)_i + (\Delta \lambda^j)_i P \\
(\delta t^j)_i &= (\delta q^j)_i + (\delta \lambda^j)_i P \\
(\Delta q^j)_i &= (\Delta q^{j-1})_i + (\delta q^j)_i
\end{align*}$$

where

$$\begin{align*}
(\delta \lambda^j)_i &= (\delta \lambda^{j-1})_i - (\delta \lambda^j)_i \\
(R^j)_i &= (\delta \lambda^j)_i P
\end{align*}$$

(3)\hspace{1cm}(4)\hspace{1cm}(5)\hspace{1cm}(6)

The main types of arc-length-type methods used in present are given in Appendix. For the initial iterative step of each incremental step in each method, the arc constraint equation can generally be written as follows

$$\begin{align*}
(\delta q^0)^T (\delta q^0)_i + \alpha^2 (\delta \lambda^0)_i P^T P = (\Delta \lambda^0)^2
\end{align*}$$

(12)

where for cylindrical arc-length method, $\alpha^2 = 0$; for ellipsoidal arc-length method, $\alpha^2 = S_p$; for the other arc-length methods, $\alpha^2 = 1$.

In recent years, the possible operation failures of arc-length-type methods were widely discussed [5,6], and the most possible failures would happen in the following cases:

(1) The constraint equation of the loading increment parameter is independent of the arc-length increment of the step, as shown in Fig.2(a). It means that this tracing method is not suitable for such kind of load-deflection path curves.

(2) The Newton linearization of the equilibrium equations or the constraint equation makes the arc-length to be independent. In some cases it means that $(\delta t^j)_i$ and $(\delta t^j)_i$ in Fig.1 have no intersection. This kind of failures occurs in all quadratic methods, as shown in Figs.2(b), (c) and (d), which means that the loading increment of the step is not suitable.

(3) In the use of all quadratic methods, unsuitable choice of the roots of constraint equations or obtaining imaginary roots from the constraint equations would result in the failure in tracing analysis, as shown in Figs.2(e) and (f). Generally speaking, this also means that the loading increment of the step is not suitable.

(4) The precision of computers and/or the precision and reliability of the used program is not suitable.

(5) The error in determining the sign of the initial loading increment parameter will lead to the tracing-back phenomenon in tracing analysis, as shown in Fig.2(g).

(6) If the structural instability has bifurcation points, most arc-length methods will not be convergent, as shown in Fig.2(h). Even if the analysis traces