A SUPPLEMENTARY STUDY OF ANISOTROPIC PLASTIC FIELDS AT A RAPIDLY PROPAGATING PLANE-STRESS CRACK-TIP (I)

Lin Baisong (林拜松)

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Abstract

The results in Ref. [1] are not suitable for the cases of $\beta \geq 2$. For this reason, by using the methods in Ref. [1] and Ref. [2], we derive the general expressions of anisotropic plastic fields at a rapidly propagating plane-stress crack-tip for both the cases of $\beta = 2$ and $\beta > 2$.

Key words rapid propagation, plane-stress, crack-tip, anisotropic plastic fields, plastic zone, general expressions

I. Introduction

We know that the results in Ref. [1] are not suitable for the cases of $\beta \geq 2$. For this reason, we use the methods in Ref. [1] and Ref. [2] to study the anisotropic plastic fields at a rapidly propagating plane-stress crack-tip for the cases of $\beta \geq 2$. All the stress components at a rapidly propagating crack-tip in an elastic perfectly-plastic material are the functions of $\theta$ only. By using this condition and the equations of steady-state motion, Hill yield conditions for both the cases of $\beta = 2$ and $\beta > 2$ and elastic-plastic constitutitional equations, we derive the general equations of anisotropic plastic fields at a rapidly propagating plane-stress crack-tip for both the cases of $\beta = 2$ and $\beta > 2$. Applying these general expressions to two particular cases of anisotropic plasticity, the general expressions of anisotropic plastic fields at a rapidly propagating plane-stress crack-tip for the two particular cases of both the cases of $\beta = 2$ and $\beta > 2$.

Fig. 1

Fig. 1 shows that the geometry of a plane-stress crack-tip which propagates rapidly along the crack-line. $(x_1, y_1, z_1)$ and $(x, y, z)$ are the stationary and the moving coordinate systems, respectively. The moving coordinate system has its origin at the rapidly propagating plane-stress crack-tip. Let the speed of the crack-tip be $c = \text{const}$. Assuming that the crack is in the steady-state motion, then the following relations:

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1 Central-South University of Technology, Changsha 430083, P. R. China
\[
\frac{\partial}{\partial t} = -c \frac{\partial}{\partial \mathbf{x}}, \quad \frac{\partial^2}{\partial t^2} = c^2 \frac{\partial^2}{\partial \mathbf{x}^2}
\]  

are obtained. From now on, we take

\[\alpha = c/\sqrt{\mu/\rho} \leq 1\]

where \(c = \sqrt{\mu/\rho}\) defines the speed of shear waves in an elastic solid with shear modulus \(\mu\) and mass density \(\rho\).

II. General Expressions for the Case of \(\beta = 2\)

For the case of \(\beta = 2\), in the moving coordinate system \(Oxyz\), we have the following system of partial differential equation:

\[
\begin{align*}
X \frac{\partial \sigma_x}{\partial x} + X \frac{\partial \sigma_y}{\partial y} + T \frac{\partial \sigma_{xy}}{\partial y} - \rho c^2 \frac{\partial u_x}{\partial x} & = 0 \\
Y \frac{\partial \sigma_y}{\partial y} - Y \frac{\partial \sigma_x}{\partial x} + T \frac{\partial \sigma_{xy}}{\partial x} - \rho c^2 \frac{\partial u_y}{\partial y} & = 0 \\
X \frac{\partial u_x}{\partial x} + Y \frac{\partial u_y}{\partial y} - \left( D_1 \frac{\partial \sigma_x}{\partial x} + D_2 \frac{\partial \sigma_y}{\partial y} \right) & = 0 \\
X \sigma_{xy} \frac{\partial \sigma_{xy}}{\partial x} - 4 \sigma_{xy} \frac{\partial u_y}{\partial y} - Y \sigma_{xy} \frac{\partial u_x}{\partial x} - 4 \sigma_{xy} \frac{\partial u_y}{\partial y} & = 0
\end{align*}
\]

where

\[
\sigma_x = \frac{1}{2} \left( \sigma_x + \sigma_y \right), \quad \sigma_y = \frac{1}{2} \left( \sigma_x - \sigma_y \right), \quad \sigma_{xy} = \frac{\sigma_{xy}}{\mu}
\]

\[
D_1 = \frac{X^2 + Y^2 - 2\nu XY}{E}, \quad D_2 = \frac{X^2 - Y^2}{E}, \quad D_3 = \frac{X^2 + Y^2 + 2\nu XY}{E}, \quad D_4 = \frac{\nu^2}{E}
\]

The Hill yield condition for the case of \(\beta = 2\) is

\[
4\sigma_x^2 + \sigma_{xy}^2 = 1
\]

If we take

\[
\sigma_- = -\frac{1}{2} \cos \varphi, \quad \sigma_{xy} = \sin \varphi
\]

Then (2.4) is identically satisfied. Where \(\varphi\) is the function of \(\theta\) only.

We know that \(\sigma_x, \sigma_y, \sigma_{xy}, u_x\) and \(v_y\) are the functions of \(\theta\) only. By substituting (2.5) into (2.1) and using the following transformation:

\[
\frac{\partial}{\partial x} = -\sin \theta \frac{d}{d\theta}, \quad \frac{\partial}{\partial y} = -\cos \theta \frac{d}{d\theta}
\]