A SUPPLEMENTARY STUDY OF ANISOTROPIC PLASTIC FIELDS
AT A RAPIDLY PROPAGATING PLANE-STRESS CRACK-TIP (I)

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Abstract

The results in Ref. [1] are not suitable for the cases of \( \beta \geq 2 \). For this reason, by using the methods in Ref. [1] and Ref. [2], we derive the general expressions of anisotropic plastic fields at a rapidly propagating plane-stress crack-tip for both the cases of \( \beta = 2 \) and \( \beta > 2 \).

Key words rapid propagation, plane-stress, crack-tip, anisotropic plastic fields, plastic zone, general expressions

I. Introduction

We know that the results in Ref. [1] are not suitable for the cases of \( \beta \geq 2 \). For this reason, we use the methods in Ref. [1] and Ref. [2] to study the anisotropic plastic fields at a rapidly propagating plane-stress crack-tip for the cases of \( \beta \geq 2 \). All the stress components at a rapidly propagating crack-tip in an elastic perfectly-plastic material are the functions of \( \theta \) only. By using this condition and the equations of steady-state motion, Hill yield conditions for both the cases of \( \beta = 2 \) and \( \beta > 2 \) and elastic-plastic constitutional equations, we derive the general equations of anisotropic plastic fields at a rapidly propagating plane-stress crack-tip for both the cases of \( \beta = 2 \) and \( \beta > 2 \). Applying these general expressions to two particular cases of anisotropic plasticity, the general expressions of anisotropic plastic fields at a rapidly propagating plane-stress crack-tip for the two particular cases of both the cases of \( \beta = 2 \) and \( \beta > 2 \).

Fig. 1

Fig. 1 shows that the geometry of a plane-stress crack-tip which propagates rapidly along the crack-line. \((x_1,y_1,z_1)\) and \((x,y,z)\) are the stationary and the moving coordinate systems, respectively. The moving coordinate system has its origin at the rapidly propagating plane-stress crack-tip. Let the speed of the crack-tip be \( c = \text{const} \). Assuming that the crack is in the steady-state motion, then the following relations:

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are obtained. From now on, we take
\[ \alpha = \frac{c}{\sqrt{\mu/\rho}} \leq 1 \] (1.2)
where \( c = \sqrt{\mu/\rho} \) defines the speed of shear waves in an elastic solid with shear modulus \( \mu \) and mass density \( \rho \).

II. General Expressions for the Case of \( \beta = 2 \)

For the case of \( \beta = 2 \), in the moving coordinate system \( Oxyz \), we have the following system of partial differential equation:

\[
\begin{align*}
X \frac{\partial \sigma_x}{\partial x} + X \frac{\partial \sigma_x}{\partial y} + T \frac{\partial \sigma_{xy}}{\partial x} &= \rho c^2 \frac{\partial u_x}{\partial x} = 0 \\
Y \frac{\partial \sigma_x}{\partial y} + Y \frac{\partial \sigma_x}{\partial y} + T \frac{\partial \sigma_{xy}}{\partial x} &= \rho c^2 \frac{\partial u_x}{\partial x} = 0 \\
X \frac{\partial u_x}{\partial x} + Y \frac{\partial u_x}{\partial y} - \left( D_1 \frac{\partial \sigma_x}{\partial x} + D_2 \frac{\partial \sigma_x}{\partial x} \right) &= 0 \\
X \frac{\partial \sigma_{xy}}{\partial x} - 4T \sigma_x \frac{\partial u_x}{\partial y} - Y \frac{\partial u_y}{\partial y} - 4T \sigma_x \frac{\partial u_x}{\partial x} &= 0 \\
-X \frac{\partial \sigma_{xy}}{\partial x} + D_3 \frac{\partial \sigma_x}{\partial x} + D_4 \frac{\partial \sigma_{xy}}{\partial x} &= 0
\end{align*}
\] (2.1)

where
\[
\sigma_x = \frac{1}{2} \left( \frac{\sigma_x + \sigma_y}{X} \right), \quad \sigma_y = \frac{1}{2} \left( \frac{\sigma_x - \sigma_y}{Y} \right), \quad \sigma_{xy} = \frac{\tau_{xy}}{Y} \] (2.2)
\[
D_1 = \frac{X^2 + Y^2 - 2\nu XY}{E}, \quad D_2 = \frac{X^2 - Y^2}{E}, \quad D_3 = \frac{X^2 + Y^2 + 2\nu XY}{E}, \quad D_4 = \frac{\tau_{xy}}{\mu}
\] (2.3)

The Hill yield condition for the case of \( \beta = 2 \) is
\[ 4\sigma_x^2 + \sigma_{xy}^2 = 1 \] (2.4)
If we take
\[ \sigma_x = -\frac{1}{2} \cos \varphi, \quad \sigma_{xy} = \sin \varphi \] (2.5)
Then (2.4) is identically satisfied. Where \( \varphi \) is the function of \( \theta \) only.

We know that \( \sigma_x, \sigma_y, \sigma_{xy}, u_x \) and \( v_y \) are the functions of \( \theta \) only. By substituting (2.5) into (2.1) and using the following transformation:
\[ \frac{\partial}{\partial x} = \frac{-\sin \theta}{r} \frac{d}{d\theta}, \quad \frac{\partial}{\partial y} = \frac{-\cos \theta}{r} \frac{d}{d\theta} \] (2.6)