RECENT PROGRESS IN NONLINEAR EDDY-VISCOSITY TURBULENCE MODELING*

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ABSTRACT: This article presents recent progresses in turbulence modeling in the Unit for Turbulence Simulation in the Department of Engineering Mechanics at Tsinghua University. The main contents include: compact Non-Linear Eddy-Viscosity Model (NLEVM) based on the second-moment closure, near-wall low-Re non-linear eddy-viscosity model and curvature sensitive turbulence model. The models have been validated in a wide range of complex flow test cases and the calculated results show that the present models exhibited overall good performance.

KEY WORDS: nonlinear eddy-viscosity model, low-Re modification, curvature effects

1 INTRODUCTION

Engineering practice of fluid mechanics requires advanced turbulence models which can deal with complex turbulent flows, such as flows with complex geometry, recirculation, streamline curvature, noninertial force, compressibility, shock wave, etc. The high-performance turbulence models are desired to have rich flow physics while keep simplicity in mathematics and robustness in computation. In this article the research focus is on the following: to develop compact Non-Linear Eddy-Viscosity Model (NLEVM) based on the second-moment closure, to develop a wall-parameter-free low-Re model to account for complex geometries and to develop a curvature sensitive turbulence model.

2 COMPACT NON-LINEAR EDDY-VISCOSITY MODEL

It is well-known that the two-equations models, for instance, the k-ε or the k-ω model, offer significantly greater simplicity in mathematics and, hence, in numerical implementation for applications in solving practical flow problems than the other more elaborate second-moment closures. The Boussinesq hypothesis, together with the k-ε model formalized by Launder & Spalding[1], is generally regarded as the standard approach within the eddy-viscosity concept. Current studies have mainly been, however, focused on the nonlinear relationship between the Reynolds-stress and strain-rate/vorticity. Applications of the nonlinear eddy-viscosity models to the investigation of complex turbulent flows have displayed encouraging results in mimicking the effects of turbulence anisotropy, streamline curvature, recirculation and adverse pressure gradient. The main common feature of the nonlinear Eddy-Viscosity-Models (EVM) is the introduction of nonlinear strain-rate/vorticity elements, or integrity basis, to the basic constitutive stress-strain relation to form nonlinear stress-strain-vorticity relations. However, the degree of nonlinearity in the existing nonlinear EVM’s differs significantly from one another: quadratic, cubic, or even as high as fifth order models have been proposed. It is not clear that if there is a mathematical theory which can keep the nonlinearity level bounded. The question on the minimal representation of the Reynolds stresses, or the minimum degree of nonlinearity in the stress-strain-vorticity relationship had not been fully solved.

In fact, from the work of Pope[2], Gatski & Speziale[3], who analyzed the nonlinear stress-strain-
vorticity constitutive equation with the theory of rational mechanics, the degree of nonlinearity is found to relate with the dimensionality of the flow in question. For incompressible two-dimensional plane flows, irrespective of the flow complexity, nonlinearity can only go to quadratic level with only three integrity basis, terms with orders high than quadratic can be reduced, with the help of the generalized Cayley-Hamilton theory\[4\], and absorbed in the coefficients of the nonlinear expansion of the constitutive equation. The nonlinear EVM’s with quadratic order had been shown to be able to capture the turbulence anisotropy in the fully-developed square duct flow although the flow is three-dimensional. But to better resolve the behavior of three-dimensional flows, or to appropriately rectify the weaknesses in the linear EVM, the introduction of the nonlinear terms at quadratic level only proved insufficient. As noted by Launder\[5\], after doing extensive studies on nonlinear EVM: Only by proceeding to cubic level does one find sufficient variety in the stress-strain couplings to achieve the desired effects. Indeed, with the cubic terms in the stress-strain relation, Craft et al.\[6\] were able to appropriately mimic swirling flow in a pipe and the axisymmetrical impinging jet with predictive model quality similar to that with any of the tested second-moment closures.

For an incompressible three-dimensional flow, based on the formulation of Explicit ASM (EASM), showed that the general relationship of the Reynolds stresses with strain rate and vorticity is a nonlinear expression with ten integrity basis. The nonlinear stress-strain-vorticity relations formulated by Shih et al.\[7\] with 18 terms are obviously in excess. In fact, from the recent work on rational mechanics the complete nonlinear stress-strain-vorticity relation requires only seven integrity basis. Such constitutive relation is certainly more compact and, hence, better for numerical implementation. However, our study will show that the minimal representation of the stress-strain-vorticity relationship requires only five integrity basis. The argument put forward here is that on the principal axes of the strain rates there are only five independent elements in the strain rate and vorticity tensors that can only give rise to only five genuinely independent integrity basis mathematically. The ten- or seven-integrity basis expressions are not the minimal form. With the application of the generalized Cayley-Hamilton theory the present work is able to reduce a five-integrity-basis stress-strain-vorticity relation from a ten-integrity-basis model.

Further, with the minimal representation of the stress-strain-vorticity relationship a compact EASM is derived. The property of this compact EASM is analyzed and discussed in a number of typical flows including irrotational distortion, two-dimensionality, swirling, etc. Finally, a new nonlinear EVM based on the present analysis is proposed\[8\]. The process of modeling can be seen in Ref. [8] and is omitted here. Finally the model can be written as:

\[
\begin{align*}
\bar{u}_i \bar{u}_j &= \frac{2}{3} \delta_{ij} k - 2 \nu_t \left\{ S_{ij} + \frac{\beta_2}{\varepsilon} \left[ S_{ik} W_{kj} + S_{jk} W_{ki} \right] - \\
&- \frac{\beta_3}{\varepsilon} \left[ \frac{S_{ij}^2}{3} - \frac{1}{3} \delta_{ij} S_{kk}^2 \right] + \frac{3 \beta_3}{2 \beta} \left( \frac{1}{\eta} - 1 \right) \frac{k}{\varepsilon} \right\} \\
&- \left[ \frac{k}{\varepsilon} \right] \left[ W_{ik} S_{kj}^2 + W_{jk} S_{ki}^2 \right] \\
\end{align*}
\]

where

\[
\begin{align*}
\nu_t &= \frac{3 \beta_1}{3 - 2 \eta^2 + 6 \xi^2} g = \left( \frac{1}{2} C_1 + \frac{P_k}{\varepsilon} - 1 \right)^{-1} \\
\beta_1 &= \left( \frac{2}{3} - \frac{C_2}{2} \right) g \\
\beta_2 &= \left( 1 - \frac{C_4}{2} \right) g \\
\beta_3 &= \left( 2 - C_3 \right) g \\
\eta^2 &= \frac{1}{8} (S \beta_3)^2 \\
\xi^2 &= \frac{1}{2} (\Omega \beta_2)^2 \\
S &= \frac{k}{\varepsilon} \sqrt{2 S_{kk}^2} \\
\Omega &= \frac{k}{\varepsilon} \sqrt{-2 W_{kk}^2} \\
\end{align*}
\]

C_1 \sim C_4 are the coefficients of pressure-strain correlations in Reynolds-stress Transport Model (RSTM).

To demonstrate that CEVM can indeed reflect the nonlinear relation in the variation of the tangential velocity W with respect to the radial distance r in the rotating pipe flow, the model performance is validated here. The selected flow case has two sets of the Reynolds and Rosby numbers: Re = 20000, Ro = 1.0 and Re = 50000, Ro = 0.5. The flow was studied experimentally by Reich et al.\[9\]. Eggels\[10\] performed LES study on this flow with slightly different Re and Ro(Re = 59500, Ro = 0.71). The typical feature of this flow from the experimental observation is the approximate quadratic relation between the tangential velocity and the radial coordinate, i.e., \(W/W_{\text{wall}} \approx \left( r/R \right)^2\) where R is the radius of the pipe.

The computational performance of CEVM is highlighted in Figs.1 and 2 showing the axial and tangential velocity profiles, respectively. In Fig.1 the axial mean flow velocity profiles show good agreement.