ON ASYMPTOTIC DECOMPOSITIONS OF \(o\)-SOLUTIONS IN THE THEORY OF QUASILINEAR SYSTEMS OF DIFFERENCE EQUATIONS

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We consider a quasilinear system of difference equations with certain conditions. We prove that there exists a formal partial \(o\)-solution of this system in the form of functional series of special type. We also prove a theorem on the asymptotic behavior of this solution.

Consider the system of difference equations

\[
\Delta y_k(t) = q_k(t) + \sum_{i=1}^{n} p_{k_i}(t) y_i(t) + \sum_{k_1+\ldots+k_n=2} p_{kk_1\ldots k_n}(t) y_1^{k_1}(t) \ldots y_n^{k_n}(t),
\]

\(t \in \mathbb{N}, \quad t \geq t_0, \quad k = 1, \ldots, n,
\]

with the conditions

(i) \(|p_{kk_1\ldots k_n}(t)| \leq AR^{-(k_1+\ldots+k_n)},\)

\(k = 1, \ldots, n, \quad k_1 + \ldots + k_n \geq 2, \quad A, R \in \mathbb{R}^+;\)

(ii) \(q_k(t) = o(1), \quad k = 1, \ldots, n, \quad t \to +\infty;\)

(iii) \(\exists P_0 = \lim_{t \to +\infty} P(t), \quad P_0 \in \mathbb{C}^{n \times n}, \quad P(t) = \left(p_{kk_1\ldots k_n}(t)\right)^n.\)

Assume that the characteristic numbers \(\lambda_k, \quad k = 1, \ldots, n,\) of the matrix \(P_0\) possess the property \(|1 + \lambda_k| \neq 0, \quad k = 1, \ldots, n.\)

Inequality (2) guarantees the absolute and uniform convergence of the series in system (1) in any domain of the form

\[
\Gamma \left\{ t \in \mathbb{N}, \quad t \geq t_0, \quad \sum_{i=1}^{n} |y_i(t)| \leq R_0 < R \right\}, \quad R_0 \in \mathbb{R}^+.
\]

Furthermore, we assume that the functions \(q_k(t), \quad p_{ki}(t),\) and \(p_{kk_1\ldots k_n}, \quad k, i = 1, \ldots, n, \quad k_1 + \ldots + k_n \geq 2,\) admit, in a certain sense (see Definitions 6 and 7), formal expansions into series of the form

\[
\sum_{k_1+\ldots+k_p=0} c_{k_1k_2\ldots k_p} f_1^{k_1}(t) f_2^{k_2}(t) \ldots f_p^{k_p}(t), \quad c_{k_1k_2\ldots k_p} \in \mathbb{C},
\]

where \( f_k(t), k = 1, \ldots, p, \) is a fixed set of functions such that

\[
\Delta^i f_k(t) = o(1), \quad \Delta^0 f_k(t) \overset{\text{def}}{=} f_k(t), \quad k = 1, \ldots, p, \quad i = 0, 1, 2, \ldots.
\]

In what follows, we denote the set of functions \( f_k(t), k = 1, \ldots, p, \) by \( (f) \).

We rewrite system (1) as

\[
\Delta Y(t) = Q(t) + P(t)Y(t) + \Psi(t, Y(t)).
\]

Consider the series

\[
\sum_{s=0}^{\infty} c_s \sigma_s(t), \quad \sigma_s(t) = \prod_{i=0}^{s-1} \left( \Delta^i f_1(t) \right)^{k_i} \ldots \prod_{i=0}^{s-1} \left( \Delta^i f_p(t) \right)^{l_i},
\]

where \( k = k_0 \ldots k_{s-1} \ldots l_0 \ldots l_{s-1} \) and, for any fixed value \( s, \) the exponents \( k_0, \ldots, k_{s-1}, \ldots, l_0, \ldots, l_{s-1} \) can be integer nonnegative numbers satisfying the condition

\[
k_0 + 2k_1 + \ldots + sk_{s-1} + \ldots + l_0 + 2l_1 + \ldots + sl_{s-1} = s.
\]

The coefficients \( c_k \) are columns of the same dimension \( n. \) The numbers \( s \) are called the orders of the corresponding terms in (5).

**Definition 1.** A vector function \( \varphi(t), t \in N, t \geq t_0, \) which is a finite sum of the type

\[
\varphi(t) = \sum_{s=s_0}^{\infty} c_s^* \sigma_s(t), \quad c_s^* \in \mathbb{C}^{n \times 1}, \quad k = k_0 \ldots k_{s-1} \ldots l_0 \ldots l_{s-1},
\]

where the terms have the same order \( s_0, \) is called a function of order \( s_0, \) which is denoted as follows:

\[
\Pi(\varphi(t)) = s_0.
\]

**Property 1.** If \( \Pi(\varphi(t)) = s_0 \) and \( c \in \mathbb{C}, \) then \( \Pi(c \varphi(t)) = s_0. \)

**Property 2.** If \( \Pi(\varphi_1(t)) = s_0 \) and \( \Pi(\varphi_2(t)) = s_0, \) then

\[
\Pi(\varphi_1(t) + \varphi_2(t)) = s_0.
\]

**Property 3.** If \( \Pi(\varphi_1(t)) = s_1 \) and \( \Pi(\varphi_2(t)) = s_2, \) then

\[
\Pi(\varphi_1(t) \varphi_2(t)) = s_1 + s_2.
\]

**Definition 2.** The following series are called, respectively, the sum, difference, and product of two formal series \( \sum_{s=0}^{\infty} W_s \) and \( \sum_{s=0}^{\infty} V_s \) of type (5):

\[
\sum_{s=0}^{\infty} (W_s + V_s), \quad \sum_{s=0}^{\infty} (W_s - V_s), \quad \sum_{s=0}^{\infty} (W_sV_s + W_{s-1} + \ldots + W_0V_0).
\]