DECAY RATES IN A BIMATERIAL CIRCULAR CYLINDER

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ABSTRACT: Decay rates in a bimaterial circular cylinder under axisymmetric torsion loading are considered via an eigen-expansion near the end of the cylinder. The decay rates depend on the shear modulus ratio of the materials and the radius ratio of inner and outer cylinders. Following the derivation of the traditional Saint-Venant end effect of an isotropic bimaterial cylinder, cases of anisotropic material (transversely isotropic material) and non-traditional Saint-Venant end effect (displacement prescribed on the side surface) are considered. This study sheds some light on the decay studies for other geometric configurations and the deformation modes of composite structures.

KEY WORDS: bimaterial circular cylinder, decay rates, Saint-Venant principle

I. INTRODUCTION

Decay analysis is a mathematical foundation of the well-known Saint-Venant principle which is considered as one of the basic tenets of elasticity theory. The applicability of beam, plate, and shell theories is highly dependent on this principle. Because the Saint-Venant principle was proposed over a century ago, the original statement is apparently too ambiguous for modern structures and materials. The development of an understanding of the principle on the basis of a rigorous mathematical formation started in the 1960's. Since then, the establishment of the Saint-Venant principle has been an active topic of research. Use of the mathematical form of the Saint-Venant’s principle, presents no conceptual and descriptive difficulties when defining the end effects in modern materials, such as highly anisotropic composites. A thorough review of the research progress on the principle was given by Horgan and Knowles[1], with a follow-up paper by Horgan[2].

Use of decay rate analysis allows one to evaluate the foundations of beam, plate and shell theories existing in the literature. The proper prescription of the boundary condition at the ends of the beams and edges of the plates and shells has been discussed via the application of this type of analysis. An example of a recent effort is the work initiated by Gregory and Wan[3] who analyzed the proper boundary conditions for a plane strain strip. Extension of their approach to the beam was carried out by Fan and Widera[4]. Further work along this line can be found in papers by Gregory, Wan and their co-workers[5].

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A decay rate study converts the qualitative Saint-Venant statement into a quantitative description via the use of a so-called eigen-expansion. For a composite cylinder, as shown in Fig.1, a generic field variable can be expanded as

\[ F(r, z) = F_0(r, z) + \sum_{k=1}^{\infty} C_k e^{-\lambda_k z} F_k(r) \]  

where \( F_0(r, z) \) is the so-called Saint-Venant solution, which is a non-decay term with respect to the coordinate \( z \). All terms in the summation have an exponential decay character. In other words, all terms with \( \text{Re}(\lambda) < 0 \) are not suitable for the solution state. With reference to the summation terms, the first term with the smallest value of \( \text{Re}(\lambda) \), has the slowest decay. One generally refers to this value of \( \lambda \) as the decay rate \( \lambda^* \), from which the decay length of end effects can be determined.

In this paper, the decay rate in a bimaterial circular cylinder under axisymmetric torsion loading is studied via the eigen-expansion Eq. (1). It is noted that the stress and deformation analysis of a homogeneous circular cylinder has long been cited in the literature because of its importance in engineering applications and because of the mathematical simplicity involved compared to other cross-section shapes. The present formulation for a composite cylinder benefits greatly from the previous studies. The detailed historical review of the subject can be found in papers by Horgan and Knowles\[^{[1]}\] and Horgan\[^{[2]}\]. More recent contributions to this subject are the papers by Stephen and Wang\[^{[6]}\] and Fan and Widera\[^{[4]}\], among others. On the other hand, results for the composite circular cylinder have not been found in the literature.

II. DECAY ANALYSIS

The coordinates shown in Fig.1 are normalized with the outer radius of the cylinder. An axisymmetric torsional deformation is assumed to exist in the bimaterial composite cylinder. The non-zero stress and displacement components are \( \sigma_{r\theta}, \sigma_{z\theta} \), and \( u_\theta \) respectively. Boundary conditions at the outer surface and continuity conditions at the interface between the inner and outer cylinder are

\[ \sigma_{r\theta} = 0 \quad \text{at} \quad r = 1 \]  

\[ [\sigma_{r\theta}] = 0 \quad [u_\theta] = 0 \quad \text{at} \quad r = h \]