A CRACK PERPENDICULAR TO AND TERMINATING AT A BIMATERIAL INTERFACE*

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ABSTRACT: Using dislocation simulation approach, the basic equation for a finite crack perpendicular to and terminating at a bimaterial interface is formulated. A novel expansion method is proposed for solving the problem. The complete solution to the problem, including the explicit formulae for the $T$ stresses ahead of the crack tip and the stress intensity factors are presented. The stress field characteristics are analysed in detail. It is found that normal stresses $\sigma_x$ and $\sigma_y$ ahead of the crack tip, are characterised by $Q$ fields if the crack is within a stiff material and the parameters $|p_T|$ and $|q_T|$ are very small, where $Q$ is a generalised stress intensity factor for a crack normal to and terminating at the interface. If the crack is within a weak material, the normal stresses $\sigma_x$ and $\sigma_y$ are dominated by the $Q$ field plus $T$ stress.

KEY WORDS: interface, perpendicular crack, generalised stress intensity factor, $T$ stress

1 INTRODUCTION

Many modern devices, tools and engineering structures are made of advanced materials, such as fiber or particle reinforced composites, metal/ceramics interfaces, laminated ceramics, adhesive joints etc. Interface failures are common features in the advanced materials and thin films. The design process of these components requires a better understanding of the failure mechanisms of these components. An important task is to study in detail the fracture characteristics of flaws along or perpendicular to the interface.

A crack perpendicular to a bimaterial interface has attracted the attention of many investigators. Zak and Williams[1] used the eigenfunction expansion method to analyse the stress singularity ahead of a crack tip, which is perpendicular to and terminating at the interface. Cook and Erdogan[2] used the Mellin transform method to derive the governing equation of a finite crack perpendicular to the interface and obtained the stress intensity factors. Erdogan and Biricikoglu[3] solved the problem of two bounded half planes with a crack going through the interface. Bogy[4] investigated the stress singularity of an infinite crack terminated at the interface with an arbitrary angle. Wang and Chen[5] used photoelasticity to determine the stress distribution and the stress intensity factors of a crack

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perpendicular to the interface. Lin and Mar\textsuperscript{[6]}, Ahmad\textsuperscript{[7]} and Meguid et al.\textsuperscript{[8]} used finite element to analyse cracks perpendicular to bimaterial in finite elastic body. Chen\textsuperscript{[9]} used the body force method to determine the stress intensity factors for a crack normal to and terminated at a bimaterial interface. Ståhle et al.\textsuperscript{[10,11]} investigated a crack growing towards a bimaterial interface. Their results showed that the crack can be deflected and to follow a smooth curved path.

2 FORMULATION OF THE CRACK PROBLEM

Figure 1 shows a finite crack perpendicular to and terminating at a bimaterial interface. A Cartesian coordinate system $ox$ is attached to the interface. The $x$ axis is along the interface and the $y$ axis is normal to the interface and coincident with the crack elongation direction. Both materials are isotropic and homogenous. The material I occupies the upper half plane $S_1$ and the material II occupies the lower half plane $S_2$.

2.1 Complex Potential

Stress and displacement in an elastic solid can be represented by two Muskhelishvili’s potentials

$$\begin{align*}
\sigma_x + \sigma_y &= 4\text{Re}(\Phi(z)) \\
\sigma_y - i\tau_{xy} &= \Phi(z) + \Omega(\bar{z}) + (z - \bar{z})\overline{\Phi'(z)} \\
2\mu(u_x + iu_y) &= \kappa\Phi(z) - \omega(\bar{z}) - (z - \bar{z})\overline{\Phi(z)}
\end{align*}$$

(1)

The complex potentials for an edge dislocation at $z = s$ in an infinite elastic solid can be expressed as

$$\begin{align*}
\Phi_0(z) &= \frac{B}{z - s} \\
\Omega_0(z) &= \frac{B}{z - s} + \frac{B}{(z - s)^2} \\
B &= \frac{\mu}{\pi i(\kappa + 1)}(b_x + ib_y)
\end{align*}$$

(2)

where $b_x$ and $b_y$ are the $x$- and $y$-components of the edge dislocation.

The interaction problem of an edge dislocation with a bimaterial interface was studied by Dundurs\textsuperscript{[12]} and Suo\textsuperscript{[13]}. The complex potentials are (see Suo\textsuperscript{[13]})

$$\begin{align*}
\Phi(z) &= \begin{cases} 
(1 + \Lambda_1)\Phi_0(z) & z \in S_1 \\
\Phi_0(z) + \Lambda_2\Omega_0(z) & z \in S_2
\end{cases} \\
\Omega(z) &= \begin{cases} 
\Omega_0(z) + \Lambda_1\Phi_0(z) & z \in S_1 \\
(1 + \Lambda_2)\Omega_0(z) & z \in S_2
\end{cases}
\end{align*}$$

(3)