DISPLACEMENT WAVE OF THE BLOOD VESSEL
——A MECHANICAL MODEL

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ABSTRACT: According to clinical observations and model experiment, we have developed a theoretical model and obtained its mathematical equations to investigate the displacement wave of the blood vessel. The waveform and its occurrence criterion have been obtained by using perturbation theory and numerical method. The present study suggests that the displacement wave is associated with the blood velocity waveform and the mechanical behaviour of the blood vessel, but not with the pressure waveform. The clinical criterion is in agreement with the observations.

KEY WORDS: displacement wave, blood vessel, pressure waveform, velocity waveform, perturbation theory

I. INTRODUCTION

Pulse feeling is one of the four essential elements in the traditional Chinese medical diagnosis. It is very interesting to expound the behaviour of pulse feeling by using biomechanics theory. A traditional viewpoint was that the pulses felt by doctors are arterial radial pulses which are the same as the pressure wave. In 1983, by using ultrasonic wave technique, Zhao R. showed that in the superficial arteries their radius changes are very small and the blood vessel apparently moves as a whole in a periodical way like a rope. The pulsatile period is the same as that of heart. Wu W. et al. believed that the pulses felt by doctors or transducers are this displacement wave but not the pressure wave, and the displacement wave is produced by the reaction force of the asymmetrical constraint of the arterial surrounding tissues.

Since 1985, we have observed and recorded the motion of the communis carotis arterial wall of about 6000 patients by using QFM-1000 Doppler ultrasonic blood flowmeter. We find that the arterial radial pulse exists in almost all the patients. For those patients with good arterial elasticity, the amplitude of this radial pulse is large, otherwise the amplitude is small. But the arterial displacement wave doesn’t always exist in patients, someone may have clear displacement motion, but many others may have not.

In the collapsible tube experiment, we found that blood pulsatile flow could cause the tube wall unsteady and could further lead to displacement wave under certain condition. This phenomenon not only exists in the collapsible tube but also in the arterial system. In fact, if the blood velocity is greater than the critical velocity of the blood vessel, no matter whether the blood vessel is collapsed or not, the displacement oscillation of the tube wall will occur, but in collapsible tube the critical velocity is low. Furthermore, if the arterial surrounding tissues are symmetrical constraint or even in the case that there is no constraint at all, the displacement wave will still occur. It means that the asymmetrical constraint can’t produce the displacement wave of the blood vessel.

In the present paper, according to the clinical observation and the model experiment, a mathematical model and the governing equations of arterial displacement wave are set up. By using perturbation theory and numerical method to solve the equations, the arterial displacement waveform and the criterion for the occurrence of oscillation are obtained. The results show that the displacement wave is associated with the pulsatile velocity of the blood flow and the physical properties of the arterial wall, but not with the pressure waveform. We find the criterion in

Received 14 May 1990
agreement with the clinic observation. In summary, if displacement wave occurs the pulses felt by doctors are the combination of the displacement pulses and radial pulses, and if no displacement wave occurs the pulses felt by doctors are the radial pulses which are nearly the same as pressure pulses.

II. MATHEMATICAL MODEL

According to the clinical and experimental observations, the displacement oscillation of the blood vessel is caused by the pulsatile velocity \( v(t) \) of blood in the artery and can be viewed as the destabilization of the tube wall. The artery treated in the paper is considered as a simple supported viscoelastic beam with some lumped masses which are introduced to simulate the doctor’s fingers when he feels the pulse (see Fig. 1). We utilize the Kelvin-Voigt model as its constitutive equation

\[
\sigma = E \cdot \gamma + \eta_w \cdot \dot{\gamma}
\]

where \( \sigma \) and \( \gamma \) are the stress and the strain of the blood vessel, respectively, \( E \) is the Young’s modulus, and \( \eta_w \) effective viscosity. According to the assumption of linear elasticity, the governing equations for displacement wave are obtained as

\[
EI \frac{\partial^4 y}{\partial x^4} + \eta_w \frac{\partial^5 y}{\partial x^4 \partial t} + (\rho Av^2 - \rho A\dot{x}) \frac{\partial^2 y}{\partial x^2} + 2\rho Av \frac{\partial^3 y}{\partial x \partial t} + \rho A \frac{\partial y}{\partial x} + M(x) \frac{\partial^2 y}{\partial t^2} = 0
\]

where

\[
M(x) = \rho A + m + \sum_{j=1}^{k} m_j \cdot \delta(x - x_j)
\]

and \( y(x, t) \) is the tube displacement from its equilibrium position, \( I \) is the moment of inertia, \( A \) the area of the tube cross-section, \( v \) the velocity of the blood flow, \( \rho \) the density of the blood, \( m \) the mass per unit length of tube, \( m_j \) a lumped mass in position \( x_j \), and \( \delta(x) \) is Dirac function. We can get the approximate solutions of Eq. (2) by using Galerkin method. Suppose the solution is

\[
y(x, t) = \sum_{m=1}^{N} \varphi_m(x) \cdot f_m(t)
\]

where \( \varphi_m(x) \) are the orthonormal eigenfunctions of the free vibration of a pinned-pinned beam. They satisfy the following equations

\[
\frac{d^4 \varphi_m}{dx^4} = \lambda_m \varphi_m
\]

\[
\varphi_m(0) = \varphi_m(L) = \frac{d^2 \varphi_m(0)}{dx^2} = \frac{d^2 \varphi_m(L)}{dx^2} = 0
\]

\[
m = 1, \ldots, N
\]

The eigenfunctions \( \varphi_m(x) \) are given as