ASYMMETRIC SLOSH WAVE EXCITATION 
IN A LIQUID-VAPOR INTERFACE 
UNDER MICROGRAVITY

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ABSTRACT: The dynamical behavior of fluids affected by the asymmetric gravity jitter oscillations, in particular, the effect of surface tension on partially-filled rotating fluids in a Dewar tank imposed by time-dependent directions of background reduced gravity accelerations is investigated. Results show that the greater the components of background reduced gravity in radial and circumferential directions, the greater will be the tendency toward increasing amplitude and degrees of asymmetry of the liquid-vapor interface profiles.

KEY WORDS: liquid-vapor interface, microgravity fluid mechanics, slosh wave excitation

I. INTRODUCTION

The instability of the liquid surface can be induced by the presence of longitudinal and lateral accelerations, vehicle vibration, and rotational fields of the spacecraft. Slosh waves are, thus, excited which produce high and low frequency oscillations in the liquid propellant. The sources of the residual accelerations range from the effects of the Earth's gravity gradient, atmospheric drag on the spacecraft, spacecraft attitude motions arising from machinery vibrations, thruster firings, crew motions, etc. Recent studies [1–8] suggest that the high frequency accelerations may be unimportant in comparison to the residual motions caused by low frequency accelerations.

Time-dependent dynamical behavior of partially-filled rotating fluids in reduced gravity environment were simulated by numerically solving the Navier Stokes equations subject to the initial and the boundary conditions [9–17]. Some of the steady-state formulations of interface shapes, in particular, for the interface intersecting the top wall of the cylinder, were compared with the available experiments carried out by Leslie [21] in a free-falling aircraft (KC-135). In the KC-135 experiments, the background gravity is approximately $10^{-2} g_0$ during the 30 s low gravity period.

In the spacecraft orbit around the Earth, the direction of local background gravity varies from $0^\circ$ along the axis of rotation to various directions in which three dimensional calculation shall be assumed. In this study, the effect of asymmetric gravity jitter excited
slosh waves with the direction of background gravity of 30°, 45° and 60° measured form the axis of rotation is investigated. The purpose of this study is to provide a tool for spacecraft dynamic control, in particular, the attitude control of spacecraft dynamic imbalance caused by the uneven liquid-vapor distributions due to the excitation of slosh waves, associated with asymmetric gravity jitter acceleration, induced by the oscillations at liquid-vapor interface.

II. MATHEMATICAL MODEL

The present study examines time-dependent fluid behavior, in particular, the dynamics of the interface between liquid helium and helium vapor, and the behavior of its oscillations due to gravity jitter accelerations. In this study, a time-dependent three dimensional mathematical formulation is adopted.

Consider a closed circular cylindrical Dewar of radius, a, with length, L, which is partially filled with a cryogenic liquid helium of constant density \( \rho \) and viscosity \( \mu \). Let us use cylindrical coordinates \((r, \theta, z)\), with corresponding velocity components \((u, v, w)\) and corresponding gravitational acceleration components \((g_r, g_\theta, g_z)\). The governing equations are shown as follows:

(A) Continuity Equation

\[
\frac{1}{r} \frac{\partial}{\partial r} (ru) + \frac{1}{r} \frac{\partial v}{\partial \theta} + \frac{\partial w}{\partial z} = 0
\]  

(B) Momentum Equations

\[
\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{v}{r} \frac{\partial u}{\partial \theta} - \frac{v^2}{r} + w \frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial r} + \rho g_r + \mu \left( \nabla^2 u - \frac{u}{r^2} - \frac{2}{r^2} \frac{\partial v}{\partial \theta} \right)
\]

\[
\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial r} + \frac{v}{r} \frac{\partial v}{\partial \theta} + \frac{w}{r} \frac{\partial v}{\partial z} \right) = -\frac{1}{r} \frac{\partial p}{\partial \theta} + \rho g_\theta + \mu \left( \nabla^2 v - \frac{v}{r^2} + \frac{2}{r^2} \frac{\partial u}{\partial \theta} \right)
\]

\[
\rho \left( \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial r} + \frac{v}{r} \frac{\partial w}{\partial \theta} + w \frac{\partial w}{\partial z} \right) = -\frac{\partial p}{\partial z} + \rho g_z + \mu \nabla^2 w
\]

where

\[
\nabla^2 = \frac{1}{r} \frac{\partial}{\partial r} (r \frac{\partial}{\partial r}) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}
\]

Let the profile of the interface between gaseous and liquid fluids be given by

\[
\eta(t, r, \theta, z) = 0
\]

The initial condition of the profile of the interface between gaseous and liquid fluids at \( t = t_0 \) is assigned explicitly, and is given by

\[
\eta(t = t_0, r, \theta, z) = 0
\]

A set of boundary conditions has to be supplied for solving the equations. These initial interface profiles used in this study have been given explicitly through the steady state computations made by Hung and Leslie[8] and Hung et al.[15] which were checked by the experiments carried out by Leslie[18]. These boundary conditions are as follows:

1. At the container wall, no-penetration and no-slip conditions assure that both the tangential and the normal components of the velocity along the solid walls will vanish. In the numerical calculation of bubble profiles for liquid helium and helium vapor, a constant