We study the biorthogonal Appell system and Kondrat'ev spaces in the case where the parameter of a $\mu$-exponential is perturbed by holomorphic invertible functions. The results obtained are applied to the investigation of pseudodifferential equations.

Non-Gaussian infinite-dimensional analysis was constructed in [1-3] for smooth analytic measures $\mu$ on a dual-kernel space $S'$. In [4], this was done for a wider class of nondegenerate analytic measures. In the present paper, we generalize some results obtained in [1-4] to the case of the perturbation of the argument of a $\mu$-exponential [2-4] by invertible vector functions holomorphic at zero together with inverse vector functions $\alpha$, $\beta : S_\mathbb{C} \to S_\mathbb{C}$ ($S$ is the Schwartz space and $S_\mathbb{C}$ is its complexification) with the purpose of applying them further to the solution of a certain class of pseudodifferential equations.

1. Consider the rigging $S \subset L_2(\mathbb{R}) \subset S'$, where $S$ is the Schwartz space $S = \text{pr lim}_{\mathbb{N}} \mathcal{H}_p$, $S'$ is the dual Schwartz space, $S' = \text{ind lim}_{\mathbb{N}} \mathcal{H}_p$ and $\mathcal{H}_p$ is a Hilbert space, $\mathcal{H}_{-p} = \mathcal{H}_p$ (for more details, see, e.g., [5, 6]). By $\langle \cdot, \cdot \rangle$ we denote a canonical pairing between elements of $S$ and $S'$, and $\| \cdot \|_p$ denotes the norm in $\mathcal{H}_p$.

We say that a function $G : S_\mathbb{C} \to \mathbb{C}$ is holomorphic at zero if there exists a neighborhood of zero $U \subset S_\mathbb{C}$ such that, for all $\eta \in U$, there exists a neighborhood of zero $V$ such that

$$\sum_{n=0}^{\infty} \frac{1}{n!} d^n G(\eta)(\theta)$$

converges uniformly on $V$ to a continuous function. According to [7] (see also [4]), $G$ is holomorphic at zero if and only if it is locally bounded and satisfies the condition of $G$-holomorphy, namely, there exist $p$ and $\varepsilon > 0$ such that, for all $\xi_0 \in S_\mathbb{C}$ with $|\xi_0|_p < \varepsilon$ and for all $\xi \in S_\mathbb{C}$, the function of one complex variable $\lambda \to G(\xi_0 + \lambda \xi)$ is holomorphic at $0 \in \mathbb{C}$.

By $\text{Hol}_0(S_\mathbb{C})$, we denote the algebra of germs of functions $G : S_\mathbb{C} \to \mathbb{C}$ holomorphic at zero equipped with the topology of inductive limit defined by the following family of norms: $n_{p,l}_{\mathbb{N}}(G) = \sup_{|\theta|_l \leq 2^l} |G(\theta)|$, $p, l \in \mathbb{N}$ [7, 4].

We call a vector function $\alpha : S_\mathbb{C} \to S_\mathbb{C}$ holomorphic at the point $0 \in S_\mathbb{C}$ if

\[(i)\quad \exists \ p \in \mathbb{N}, \exists \ \varepsilon > 0 : \forall \xi_0 \in S_\mathbb{C} \text{ such that } |\xi_0|_p < \varepsilon, \forall \xi \in S_\mathbb{C}, \text{ the function of one complex variable } \lambda \to \alpha(\xi_0 + \lambda \xi) \text{ is holomorphic at } 0 \in \mathbb{C}, \text{ i.e., it is differentiable in a neighborhood of zero with respect to } \lambda \text{ (in } |\cdot|_p, \forall \ p' \in \mathbb{N});\]
(ii) (the local boundedness) \( \forall p \in \mathbb{N} \exists C_p > 0: \forall \xi \in A \ | \xi|_p \leq C_p \Rightarrow \forall p' \in \mathbb{N} \exists C'_{p'} : \forall \xi \in A \ | \alpha(\xi)|_{p'} \leq C'_{p'} , A \) is an arbitrary bounded set in \( S_\xi \).

By analogy [8], it can be shown that, in this case, \( \alpha \) is continuous in a neighborhood of \( 0 \in S_\xi \). Moreover, the requirements of continuity and local boundedness in the definition of holomorphic function are mutually interchangeable.

It can be established similarly to [7, 9] that the following statement is valid:

**Lemma 1.** A function \( \alpha : S_\xi \rightarrow S_\xi \) holomorphic at zero and invertible in a neighborhood of zero can be represented as

\[
\alpha(\theta) = \sum_{k=1}^{\infty} C_k \theta^k, \quad C_k \in \mathbb{C}, \quad k \in \mathbb{N}, \quad \theta \in S_\xi,
\]

where the series converges in \( S_\xi \) for \( \max_{s \in \mathbb{R}} |\theta(s)| < \delta \) and \( \delta \) depends on \( \alpha \).

**Proof.** The proof of Lemma 1 follows easily from the pointwise representation

\[
\alpha(\theta(s)) = \sum_{k=1}^{\infty} C_k \theta(s)^k,
\]

which converges under the conditions of the lemma uniformly in \( s \), and from the fact that \( S_\xi \) is an algebra. By virtue of the last fact, all partial sums of series (1) belong to \( S_\xi \).

An analogous expansion is also valid for the inverse function:

\[
\alpha^{-1}(\theta) = \sum_{k=1}^{\infty} K_k \theta^k.
\]

For convenience, we assume that all sums of the form \( \sum_{k=1}^{0} \) are equal to one by definition.

**Lemma 2.** Assume that \( G \in Hol_0(S_\xi) \) and \( \alpha : S_\xi \rightarrow S_\xi \) is holomorphic at zero. By definition, we set \( \hat{G}(\theta) = G(\alpha(\theta)). \) Then \( \hat{G} \in Hol_0(S_\xi) \).

**Proof.** The idea of the proof is to check the local boundedness and \( G \)-holomorphy of the function \( \hat{G} \). The local boundedness is almost evident, and the \( G \)-holomorphy can easily be proved with the use of the rule of differentiation of a composite function.

2. Following [4], we consider the class of probability measures on the measurable space \( (S', \mathcal{B}(S')) \) that satisfy the following conditions:

(i) the Laplace transformation \( \mu \)

\[
l_{\mu}(\theta) := \int_{S'} \exp \{ \langle x, \theta \rangle \} \, d\mu(x), \quad \theta \in S_\xi.
\]