INVESTIGATION OF STABILIZATION OF A MATHEMATICAL MODEL OF A DYNAMICAL SYSTEM WITH RANDOM INFLUENCE IN THE RESONANCE CASE

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We construct and investigate a mathematical model of a dynamical system with random influence stabilized by increasing the frequency of random influence.

1. Statement of the Problem

The simplest mathematical model of a dynamical system with random periodic influence is described by the system of linear differential equations

\[
\frac{dX(t, \mu)}{dt} = \begin{pmatrix} 0 & 1 \\ -\omega^2 - \mu a(t, \xi(t)) & -\mu \beta \end{pmatrix} X(t, \mu),
\]

where \( \xi(t) \) is a periodic Markov random process that takes states \( \theta_1, \ldots, \theta_n \) with probabilities \( p_1, p_2, \ldots, p_n \) satisfying the system of ordinary differential equations

\[
\frac{dp_k(t)}{dt} = \sum_{s=1}^{n} \alpha_{ks}(t)p_s(t),
\]

\( \alpha_{kk}(t) \leq 0, \quad \alpha_{ks}(t) \geq 0, \quad k \neq s, \quad k, s = 1, \ldots, n, \)

and \( \sum_{s=1}^{n} \alpha_{ks}(t) \equiv 0, \quad k = 1, \ldots, n, \)

\( a(t, \xi(t)) = (a(t, \theta_1), a(t, \theta_2), \ldots, a(t, \theta_n)) \equiv (a_1(t), a_2(t), \ldots, a_n(t)), \quad a_k(t + 2\pi) = a_k, \quad k = 1, \ldots, n, \)

and \( \mu \) is a parameter, \( \mu > 0 \).

It is very important for applications to study conditions for the stable mode of functioning of a dynamical system with random influence, in particular, in the resonance case.

From the mathematical point of view, the problem is posed as follows: It is necessary to establish conditions for stability of random solutions of system (1) in the mean square.

2. Derivation of Moment Equations

Let us clarify the idea of the solution of the problem posed above by considering the special case of the system of equations (1) with a random process \( \xi(t) \) that takes two states \( \theta_1 \) and \( \theta_2 \) with probabilities \( p_1 \) and \( p_2 \) satisfying the system of equations

and the random variable \( a(t, \xi(t)) \) is defined as follows:

\[
a(t, \xi(t)) = \begin{cases} 
\alpha + \cos 2\omega t, & \xi(t) = \theta_1, \\
\alpha - \cos 2\omega t, & \xi(t) = \theta_2,
\end{cases}
\]

We apply the method of moments, beginning with the general system of moment equations

\[
\begin{align*}
\frac{dD_1}{dt} &= A_1 D_1 + D_1 A_1^* - \nu D_1 + \nu D_2, \\
\frac{dD_2}{dt} &= A_2 D_2 + D_2 A_2^* + \nu D_1 - \nu D_2,
\end{align*}
\]

where \( A_1 \) and \( A_2 \) are the matrices of coefficients of the system of equations (1) for the values of the random variable \( a(t, \xi(t)) \) equal to \( a_1(t) \) and \( a_2(t) \), respectively.

By the changing the variables according to the relations

\[
D \equiv \frac{D_1 + D_2}{2}, \quad Q \equiv \frac{D_1 - D_2}{2},
\]

\[
D \equiv \begin{pmatrix} d_1 & d_2 \\
d_2 & d_3 \end{pmatrix}, \quad Q \equiv \begin{pmatrix} q_1 & q_2 \\
q_2 & q_3 \end{pmatrix},
\]

\[
\gamma_1 \equiv \mu \alpha + \omega^2, \quad \gamma_2 \equiv \mu \beta + \nu, \quad \gamma_3 \equiv \gamma_2 + \nu,
\]

we reduce the system of equations (3) to the following system of differential equations with periodic coefficients [2]:

\[
\begin{align*}
\frac{d^3 d_1}{dt^3} + 3 \mu \beta \frac{d^2 q_1}{dt^2} + 2 (2 \gamma_1 + \mu \beta^2) \frac{dq_1}{dt} + 4 \mu \beta \gamma_1 d_1 \\
+ 4 \mu \cos 2 \omega t \frac{dq_1}{dt} + 4 \mu (\gamma_2 \cos 2 \omega t - \omega \sin 2 \omega t) q_1 &= 0, \\
\frac{d^3 q_1}{dt^3} + 3 \gamma_3 \frac{d^2 q_1}{dt^2} + 2 (2 \gamma_1 + 2 \nu \gamma_2 + \gamma_3^2) \frac{dq_1}{dt} + 4 \gamma_3 (\gamma_1 + \gamma_2 \nu) q_1 \\
+ 4 \mu \cos 2 \omega t \frac{dq_1}{dt} + 4 \mu (\gamma_2 \cos 2 \omega t - \omega \sin 2 \omega t) d_1 &= 0.
\end{align*}
\]